# A REAL OPTIONS DESIGN FOR QUALITY CONTROL CHARTS

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**ABSTRACT**

We develop a financial model for a manufacturing process where quality can be affected by an assignable cause. We value the real options associated with applying a statistical process control chart using the Black-Scholes equation, binomial and pentanomial lattices, and Monte Carlo simulation methods. This valuation gives decision makers a way to choose the appropriate quality control strategy based on an integrated view of the market dynamics with the manufacturing operational aspects. An industry case is used to demonstrate the application of real options to value control chart decisions. Web based programs are given to value the alternatives in the case study, making the valuation task accessible to other users.

## INTRODUCTION

In the forward to *Real Options* (Trigeorgis (1999)), Scott Mason writes: "Flexibility has value. While this statement is obvious at the conceptual level, it is surprisingly subtle at the applied level." The question then becomes: Precisely how valuable is flexibility? The financial arena was the original ground for the application of the options-based framework to the valuation of flexibility. More recently, managerial operating flexibility has been likened to financial options. The goal of our research is to view the flexibility surrounding manufacturing operations using financial options. Nembhard, Shi, and Park (2000) develop a framework for the broad scope of this research activity.

In this paper, we specifically consider the manufacturing decision to introduce statistical process control (SPC) charts to monitor quality. Precise methods to design control charts that minimize the cost or maximize the profit of a process have been proposed by a number of authors. These methods yield control chart designs known as economic designs (e.g., see Collani et al. (1994) and Lorenzen and Vance (1986)). They result in models that help determine the control chart parameters that will best suit a process. However, they do not account for the dynamic market conditions that have effect on the decisions manufacturers will make and hence the profitability of the manufacturing operation. The need for this approach was motivated by the inability of existing economic control chart methods to address dynamics in the market conditions.

We use the options approach to find the value of applying an SPC chart during a specified length of time, considering future uncertain market variables. The problem is analyzed as European and American options using the Black-Scholes equation, binomial and multinomial lattices, and Monte Carlo simulation for an industry case study. Results of the approaches are compared with numerical examples. Using the proposed design, a company will be able to answer questions about the long-term value of implementing control charts. This will go beyond the traditional statements such as "SPC improves quality" or "the process now produces less scrap" to an ability to determine the bottom-line dollar value to the organization that can be brought about by using (or not using) control charts.

## OPTION MODELS

Fundamentally, an option is the right, but not the obligation, to take an action in the future (Amram and Kulatilaka (1999)). Sometimes, options are associated with investment opportunities that are not financial instruments. These operational options are often termed *real options* to emphasize that they involve real activities or real commodities, as opposed to purely financial commodities, such as stock options (Luenberger (1998)).

A European option gives a holder the right to exercise the option on the expiration date. First, we formulate the control chart problem as a series of European options that expire at different time points. In our context, this means that the control chart can be used or not used (that is the option) in any time period, which are the expiration dates of the options.

An American option gives a holder the right to exercise the option on *or before* the expiration date, i.e., an American option gives the opportunity for early exercise. We also formulate the control chart problem as an American option. In this formulation, once a decision is made, control charts are used until the expiration date.

In most manufacturing systems, there are multiple sources of uncertainty. Valuing real options for such an environment requires the analysis of projects whose values depend on multiple state variables. TABLE 1 gives the option valuation approaches that may be used depending on the number of variables in the problem, and the type of the option (European or American).

In the case of just one state variable, the binomial lattice approach of Cox, Ross, and Rubinstein (CRR) (1979) can be used to value both European and American options. The Black-Scholes method (1973) can be used to value a European option for one state variable. Since Black-Scholes valuation is an analytic solution for the option, it yields a precise result. The binomial lattice provides a numerical solution; it yields an approximate result that converges to the Black-Scholes valuation as the number of steps in the lattice increases.

Boyle (1988) developed an extension of the CRR procedure for option valuation in the case of two state variables. Boyle, Evnine, and Gibbs (BEG) (1989) developed an *n*-dimensional extension of the CRR procedure. Kamrad and Ritchken (KR) (1991) developed a similar technique for valuing projects for one or more state variables. When valuing options with two state variables, the KR approach uses a pentanomial lattice. Monte Carlo simulation provides a good alternative for valuing European options with one or more state variables.

**TABLE 1**. Approaches to Use Depending on Option Type and Number of Variables

| | **One variable** | **Two variables** |
|---|---|---|
| **European** | Black-Scholes Equation<br>Binomial Lattice<br>Monte Carlo Simulation | Pentanomial Lattice<br>Monte Carlo Simulation |
| **American** | Binomial Lattice | Pentanomial Lattice |

## CONTROL CHARTS

Control charts are used to keep a process in statistical control, where the output quality is at a target level. Deming (1986) explains that a process can be disturbed by common causes and special causes. Common causes are associated with the usual steady-state running of the process when it is in a state of control. Special causes may be thought of as problems that arise periodically in a somewhat unpredictable fashion. Control charts help find the special causes, eliminate them, and return the process to its target level.

If a control chart is not used, the manufacturer may not be aware that the system is producing low quality parts. These parts may be detected through other means, such as final inspection, and be scrapped. If these parts reach consumers, they may be returned. Even if they are not returned, the manufacturer may get a reputation for bad quality, and this may reduce the demand and cause a loss of market share. For these reasons, there is a cost of not using control charts to keep the process in statistical control. Using control charts has a cost, too. One time costs include equipment and software. Ongoing costs include the time for an operator to take the sample, assess the sample performance, and record the observation. Of course there is additional cost if the sample must be destroyed in the testing process. The fundamentals of control charts and SPC are addressed in several introductory texts such as Montgomery (2000) and Grant and Leavenworth (1996).

We consider the $\overline{X}$ control chart here because it is the standard "workhorse" in many SPC applications. In our problem formulation, there is an option to use the $\overline{X}$ control charts. We evaluate the option value of using control charts with a decision criterion to maximize the profit in light of market dynamics such as price and number of sales.

### ORGANIZATION

The remainder of this paper is organized as follows. First, we define the financial model that will be used to find the option value of control charts. Next, we discuss the Black-Scholes equation, binomial and pentanomial lattice approaches, and Monte Carlo simulation procedure that we use to value the option. Then we demonstrate our modeling approach with an industry case study, and present the difference between option valuation and net present value (NPV) analysis. We also give the web based computer programs and their results. Next we discuss how the results can be used for decision-making. Finally, we provide our concluding thoughts and plans for future work.

## APPLYING FINANCIAL MODELS TO MANUFACTURING QUALITY CONTROL

In this section, we provide a framework for the financial model that will be used in this paper. We begin with a model that has only one state variable, then continue with a model that has two state variables. We also discuss the relationship between the revenue variables, control chart effect, profit, and option value.

### FINANCIAL MODEL WITH ONE VARIABLE

Total sales revenue of a product is affected by two main sources: the number of sales and the price of the product. Our first model has one state variable, which is the number of sales. Let $S_1(t)$ be the number of sales for the product during the time interval beginning at time $t$. We assume that the price of the product is constant. Let $S_2$ be the price of the product. Then, total sales revenue $R(t)$ of the product per time interval that begins at time $t$ is

$$R(t) = S_1(t)S_2.$$

Assuming that the number of sales is equal to the number of units produced for each time interval, the total profit $P(t)$ per time interval that begins at time $t$ can be defined as

$$P(t) = R(t) - F - S_1(t)V.$$

where $F$ is the fixed production cost per time interval and $V$ is the variable production cost per unit product.

Let $g$ be the loss from not applying $\overline{X}$ control charts per unit product, represented as a fraction of revenue (due to production scrap, product returns, loss of market share, etc.). Let $K$ be the cost of using $\overline{X}$ control charts per time interval, and let it be constant over all time intervals. Then, the profit per time interval can be represented as

$$P(t) = (1 - g)S_1(t)S_2 - F - S_1(t)V \quad \text{without } \overline{X} \text{ chart} \tag{1a}$$

or

$$P(t) = S_1(t)S_2 - F - S_1(t)V - K \quad \text{with } \overline{X} \text{ chart.} \tag{1b}$$

Let $S(t)$ be the revenue gain when $\overline{X}$ control charts are used. Then, the profit gain $D(t)$ is the difference between Eq. (1b) and Eq. (1a), which reduces to

$$D(t) = gS_1(t)S_2 - K = S(t) - K. \tag{2}$$

### FINANCIAL MODEL WITH TWO VARIABLES

Now let us define the model for two state variables. As before, let $S_1(t)$ be the number of sales for the product during the time interval that begins at time $t$. Now, instead of assuming the price of the product is constant, we define it as a variable. Let $S_2(t)$ be the price of the product during the time interval that begins at time $t$. Then, total sales revenue of the product during the time interval that begins at time $t$ is

$$R(t) = S_1(t)S_2(t).$$

With two variables, the profit per time interval can be represented as

$$P(t) = (1 - g)S_1(t)S_2(t) - F - S_1(t)V \quad \text{without } \overline{X} \text{ chart} \tag{3a}$$

or

$$P(t) = S_1(t)S_2(t) - F - S_1(t)V - K \quad \text{with } \overline{X} \text{ chart.} \tag{3b}$$

FIGURE 1 shows the relationship between the variables (number of sales and price), control chart effect, profit, and option value. Profit is affected by the price of the product and the number of sales. Profit also depends on whether the control chart is used or not. The option value is found by evaluating the extra profit that may be possible by using the charts in the future when it is favorable to do so. Depending on the option value, we decide whether to use the control charts now, possibly later, or never.

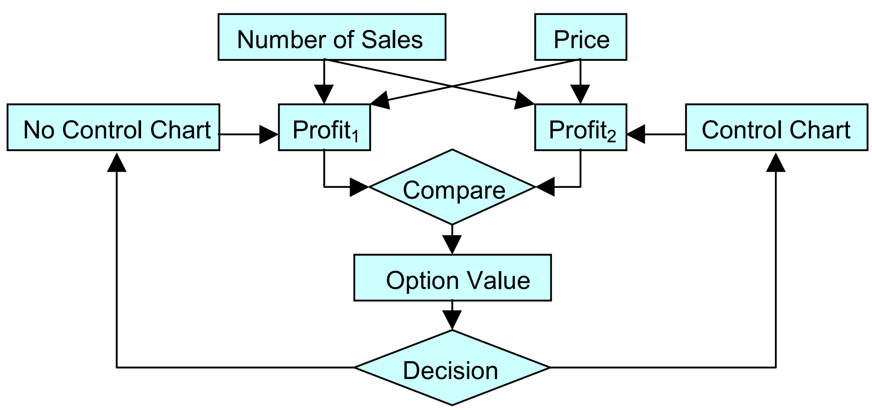


**FIGURE 1**. Relationship between the variables, control chart effect, option value and the decision.

## OPTION VALUATION MODELS

### EUROPEAN OPTION VALUATION MODELS

To value the control chart option with one variable, as given in Eq. (1a) and (1b), we will use three procedures: Black-Scholes valuation; binomial lattice approach; and Monte Carlo simulation. The Black-Scholes equation has the advantage of yielding a precise analytic solution. The binomial lattice approach gives a visual layout of the state variable values for all jumps at each time interval. Monte Carlo simulation has the advantages of easy application, of providing an estimate of the variability of the option value, and of identifying the maximum, average, and minimum profits from the simulation.

To value the control chart option with two variables, as given in Eq. (3a) and (3b), we will use pentanomial lattice and Monte Carlo simulation.

There are five parameters that determine the value of a European call option on a share of stock. TABLE 2 shows how to correlate the parameters of the control chart option with the five parameters of a stock option.

**TABLE 2**. Interpretation of Control Chart Option Parameters

| Control Chart Option | European Call Option | Parameter |
|---|---|---|
| Profit gained using control charts | Stock price | $S$ |
| Cost of using control charts | Exercise price | $K$ |
| Length of time to use control charts | Time to expiration | $T$ |
| Risk-free rate of interest | Risk-free rate of interest | $r$ |
| Volatility for profit gained using control charts | Variance of returns on stock | $\sigma^2$ |

*Black-Scholes Formula* (*for One Variable*)

Black and Scholes (1973) show that if the interest rate $r$ is constant and continuously compounded during the time $[0, T]$ where $T$ is the expiration time of the option, then the value of a European call option $c$ is defined by

$$c(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{4a}$$

where, $N(x)$ denotes the standard cumulative normal probability distribution, and

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{4b}$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t}\ . \tag{4c}$$

In our model, $S = gS_1(0)S_2$ is the value of the variable that will be used in Eq. (4a) and (4b), where $S_1(0)$ is the initial number of sales per time interval. We define a series of European options with exercise price $K$. Each option expires at a different time and expiration times are equally spaced. Let $K$ be the cost of applying $\overline{X}$ control charts per time interval. The control chart is used if the profit gained using the control chart is larger than the cost of applying the chart, i.e., if $S(t) > K$.

*Binomial Lattice* (*for One Variable*)

Suppose that the initial value of a state variable is $S$. An up move for the state variable is denoted by $u$, and a down move is denoted by $d$. At the end of a time interval $\Delta t$, the value of the state variable will either be $uS$ or $dS$, where $u > d > 0$. Then, the value of a single-period European call option is

$$c = \frac{1}{e^{r\Delta t}}\left(\frac{e^{r\Delta t} - d}{u-d}c_u + \frac{u - e^{r\Delta t}}{u-d}c_d\right) \tag{5a}$$

where $c_u$ is the value of the call option if the state variable value is $uS$ at the end of the time interval $\Delta t$, and $c_d$ is the value of the call option if the state variable value is $dS$ at the end of the time interval $\Delta t$. The $u$ and $d$ values are calculated as

$$u = e^{\sigma\sqrt{\Delta t}} \tag{5b}$$

and

$$d = 1/u. \tag{5c}$$

For multiperiod options, we extend the solution method by working backward one step at a time. In our model, the initial value of the state variable is $S = gS_1(0)S_2$.

*Monte Carlo Simulation* (*for One Variable*)

Simulation models may be used to give numerous possible paths of evolution from the present to the final date of the option for underlying state variables. In the commonly used Monte Carlo simulation method, the maximum profit on each path is determined and the payoff is calculated (Amram and Kulatilaka (1999)).

Suppose that the process followed by the underlying variable $S$ in a risk-neutral world is

$$\mathrm{d}S = \mu S \mathrm{d}t + \sigma S \mathrm{d}z \tag{6}$$

where $z$ is a Wiener process, $\mu$ is the expected return in a risk-neutral world ($\mu = r$), and $\sigma$ is the volatility. To simulate the path followed by $S$, we divide the life of the underlying variable into $N$ short intervals of length $\Delta t$ and approximate Eq. (6) as

$$S(t+\Delta t) - S(t) = \mu S(t)\Delta t + \sigma S(t)\varepsilon\sqrt{\Delta t} \tag{7}$$

where $S(t)$ denotes the value of $S$ at time $t$, and $\varepsilon$ is a random sample from a normal distribution with a mean zero and unit standard deviation. This enables the value of $S$ at time $\Delta t$ to be calculated from the initial value of $S$, the value at time $2\Delta t$ to be calculated from the value at time $\Delta t$, and so on. One simulation trial involves constructing a complete path for $S$ using $N$ random samples from a normal distribution (Hull (1997)).

From Ito's lemma (see Hull (1997) for a discussion of Ito (1951)), the process followed by $\ln S$ is

$$d\ln S = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\, dz$$

so that

$$S(t+\Delta t) = S(t)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right] \tag{8}$$

Equation (8) is used to construct a path for $S$ in a similar way to Eq. (7). Whereas Eq. (7) is true only in the limit as $\Delta t$ tends to zero, Eq. (8) is exactly true for all $\Delta t$ (Hull (1997)).

In our model, we use Eq. (8) to determine the underlying variable $S(t) = gS_1(t)S_2$, the revenue gain when control charts are used during the time interval beginning at time $t$. In order to find the option value estimate, we calculate the average of $\max[(S(T) - K),0]$ values from all runs and take the present time value of that average.

*Pentanomial Lattice* (*for Two Variables*)

If we want to include two state variables in the model, the two-state KR approach (Kamrad and Ritchken (1991)) can be used to find the option value of $\overline{X}$ control charts. Recall that the two state variables are $S_1(t)$ (number of sales at time interval $t$) and $S_2(t)$ (price at time $t$).

The KR approach is as follows. Assume the joint density of the two state variables $S_1(t)$ and $S_2(t)$ is bivariate lognormal. For state variable $i$ ($i$ = 1,2), let the instantaneous mean be $\mu_i = r - \sigma_i^2/2$, where $r$ is the risk-free interest rate, and let the instantaneous variance be $\sigma_i^2$. For each state variable over $[t, t + \Delta t]$, we have

$$\ln S_i(t + \Delta t) = \ln S_i(t) + \zeta_i(t)$$

where $\zeta_i(t)$ is a normal random variable with mean $\mu_i\Delta t$ and variance $\sigma_i^2\Delta t$. Correlation between $\zeta_1(t)$ and $\zeta_2(t)$ is $\rho$.

The joint normal random variable $\{\zeta_1(t), \zeta_2(t)\}$ is approximated by a pair of multinomial discrete random variables with the following distribution:

| $\zeta_1(t)$ | $\zeta_2(t)$ | Probability |
|---|---|---|
| $\upsilon_1$ | $\upsilon_2$ | $p_1$ |
| $\upsilon_1$ | $-\upsilon_2$ | $p_2$ |
| $-\upsilon_1$ | $-\upsilon_2$ | $p_3$ |
| $-\upsilon_1$ | $\upsilon_2$ | $p_4$ |
| 0 | 0 | $p_5$ |

where $\upsilon_i = \lambda\sigma_i\sqrt{\Delta t}$ ($i$ = 1,2) and $\lambda \geq 1$.

The convergence of the approximating distribution to the true distribution as $\Delta t \to 0$ is ensured by setting the first two moments of the approximating distribution to the true moments of the continuous distribution. Specifically, this means

$$E\{\zeta_1(t)\} = \upsilon_1(p_1 + p_2 - p_3 - p_4) = \mu_1\Delta t,$$
$$E\{\zeta_2(t)\} = \upsilon_2(p_1 - p_2 - p_3 + p_4) = \mu_2\Delta t,$$

$$\text{Var}\ \{\zeta_1(t)\} = \upsilon_1^2\ (p_1 + p_2 + p_3 + p_4) = \sigma_1^2\Delta t + O(\Delta t), \text{ and}$$

$$\text{Var}\ \{\zeta_2(t))\} = \upsilon_2^2\ (p_1 + p_2 + p_3 + p_4) = \sigma_2^2\ \Delta t + O(\Delta t).$$

In addition, the covariance terms must also be equal. This is achieved by equating the expected value of the product of the two variables,

$$\text{E}\{\zeta_1(t)\ \zeta_2(t)\} = \upsilon_1\upsilon_2\ (p_1 - p_2 + p_3 - p_4) = \sigma_1\sigma_2\rho\Delta t + O(\Delta t).$$

After substituting $\upsilon_i = \lambda\sigma_i\sqrt{\Delta t}(i = 1{,}2)$ and using a sufficiently small $\Delta t$, the above equations yield the following expressions for $p_1$, $p_2$, $p_3$, and $p_4$ :

$$p_1 = \frac{1}{4}\left[\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right] \tag{9a}$$

$$p_2 = \frac{1}{4}\left[\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right] \tag{9b}$$

$$p_3 = \frac{1}{4}\left[\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(-\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right] \tag{9c}$$

$$p_4 = \frac{1}{4}\left[\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(-\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right] \tag{9d}$$

Since $p_1 + p_2 + p_3 + p_4 + p_5 = 1$, we have

$$p_5 = 1 - \frac{1}{\lambda^2} \tag{9e}$$

There are five paths leaving each node in the pentanomial lattice. Just as in the binomial lattice, an up move for each state variable is denoted by $u$, and a down move is denoted by $d$. It is convenient to reduce the number of nodes by imposing the condition that $ud = 1$, so that an up followed by a down is equal to 1. The total number of nodes after $n$ iterations is

$$\sum_{i=0}^{n} 5^i = \left(5^{n+1} - 1\right)/4 \quad \text{if } ud \neq 1, \text{ and}$$

$$\sum_{i=0}^{n}\left[(i+1)^2+i^2\right]=\left(4n^3+12n^2+14n+6\right)/6 \quad \text{if } ud=1.$$

In just 10 iterations, a pentanomial lattice will have 12,207,031 nodes if $ud \neq 1$, whereas there will be only 891 nodes if we impose $ud = 1$.

FIGURE 2 shows the first two iterations of a pentanomial lattice with the condition $ud = 1$. The first element in parenthesis shows the change in the first state variable, and the second element shows the change in the second state variable. As can be seen in FIGURE 2, there are a total of 19 nodes in the pentanomial lattice for two iterations when $ud = 1$.

We construct a pentanomial lattice, where the last column elements are

$$\max\left[\,(gS_1(t)S_2(t)-K),\,0\,\right]. \tag{10}$$

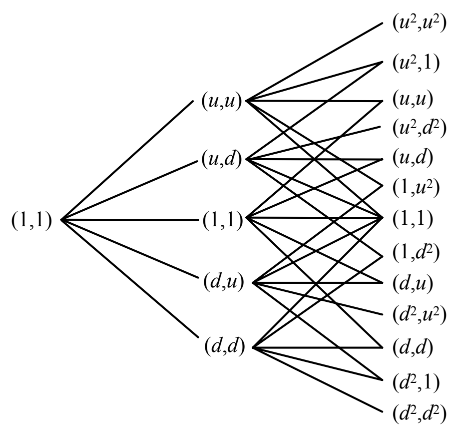


**FIGURE 2**. A pentanomial lattice with the condition $ud = 1$.

Backward discounting is applied on the lattice starting from the last time interval. First, an expected value is found by multiplying the jump probabilities (see Eq. (9)) with the corresponding profit values in the five nodes. Next, this expected value is discounted with the risk-free interest rate in one interval. This value is the expected amount of profit for the last step, which is determined from the node in the preceding time interval. The expected discounted value is added to the profit value in the origin node for the five following nodes, so that the expected total profit is found for those two time intervals. When this calculation is done for all nodes in one time interval, and then for all remaining nodes going back one time interval in each iteration, the expected discounted value at time zero is determined. The option value $c$ is equal to this discounted value.

*Monte Carlo Simulation* (*for Two Variables*)

Monte Carlo simulation for valuing an option with one underlying variable was discussed above. If we want to include both number of sales and price as variables in the model, Monte Carlo simulation can still be used.

Since the two state variables may be correlated, we need to generate correlated values for them. Two correlated samples $\varepsilon_1$ and $\varepsilon_2$ are found using a two-step procedure. First, independent samples $x_1$ and $x_2$ from a univariate standardized normal distribution are obtained by generating $U_1$ and $U_2$ as IID U(0,1), then setting $x_1 = \sqrt{-2InU_1}\cos(2\pi U_2)$ and $x_2 = \sqrt{-2InU_1}\sin(2\pi U_2)$ (Law and Kelton (2000)). Second, the required samples $\varepsilon_1$ and $\varepsilon_2$ are calculated as:

$$\varepsilon_1 = x_1$$

and

$$\varepsilon_2 = \rho x_1 + x_2\sqrt{1-\rho^2}$$

where $\rho$ is the coefficient of correlation (Hull (1997)).

AMERICAN OPTION VALUATION MODELS

In our previous European option models, we formulated the control chart problem as a series of European options, where each option expired on a different date and each had an exercise cost.

In American option models, we assume that the option will be exercised only one time, and thereafter, the decision will be applied until the expiration date. In other words, if we decide to apply the control charts at some date, then we will continue to apply them until the expiration date of the option. We assume that there will be only one exercise cost instead of a series of exercise costs. Given this, we must determine a total exercise cost for the option horizon. We let $K_A$ be the total value of the costs $K$ at the middle point between the beginning and the expiration of the option. Hence,

$$K_A = K\frac{(e^{r\Delta t})^{n+1}-1}{(e^{r\Delta t}-1)(e^{r\Delta t})^{n/2}} \tag{11}$$

where $n$ is the number of time intervals.

To value the American option for the control chart problem, we will use a binomial lattice for the one-variable problem and pentanomial lattice for the two-variable problem.

## USING REAL OPTIONS TO VALUE CONTROL CHART DECISIONS: AN INDUSTRY CASE STUDY

In order to illustrate the benefits of real options to value control chart decisions, we will use the case of HomeWindow Fashions[1]. HomeWindow manufactures several types of window blinds including horizontal and vertical blinds, and wood and fabric blinds. They are a very vertically integrated company that also produces many of the components such as the metal railing and plastic levers needed for assembly.

We focused on a part for vertical blinds that is manufactured by an extrusion process. TABLE 3 shows the empirical data for this part. Column 1 shows the month. Column 2 shows the number of sales for the part. The current number of sales for the part is $S_1(0)$ = 872,640 per month, which was obtained for the current month after the data for the past nine months. Column 3 shows the price of the part. The current price of the part is $S_2(0)$ = \$5.678. Column 4 shows the logarithm of the increase rate of number of sales for two consecutive months. Column 5 shows the logarithm of the increase rate of price for two consecutive months. Volatility is a measure for the variability of the variables. The volatility of the number of sales and price are 0.930354 and 0.059634, respectively. The yearly volatility of the number of sales and price are determined by

$$\sqrt{12} \times \sqrt{\frac{1}{8}\sum_{t=1}^{9}(m_t - \overline{m})^2} = 0.930354 \qquad (12)$$

and

$$\sqrt{12} \times \sqrt{\frac{1}{8}\sum_{t=1}^{9}(n_t - \overline{n})^2} = 0.059634 \qquad (13)$$

where $m_t = \ln[S_1(t+1)/S_1(t)]$, and $n_t = \ln[(S_2(t+1)/S_2(t)]$ (Amram and Kulatilaka (1999)).

The correlation is 0.111344, which is simply the correlation between columns 4 and 5 in TABLE 3.

**TABLE 3.** Data for the Industry Case Study

| $t$ | $S_1(t)$ | $S_2(t)$ | $\ln[S_1(t+1)S_1(t)]$ | $\ln[S_2(t+1)S_2(t)]$ |
|---|---|---|---|---|
| 1 | 231,360 | 5.5685 | | |
| 2 | 311,200 | 5.4095 | 0.296461 | -0.028969 |
| 3 | 225,600 | 5.3866 | -0.321672 | -0.004242 |
| 4 | 405,120 | 5.3952 | 0.585420 | 0.001595 |
| 5 | 490,560 | 5.3712 | 0.191364 | -0.004458 |
| 6 | 416,640 | 5.3907 | -0.163325 | 0.003624 |
| 7 | 454,080 | 5.3788 | 0.086051 | -0.002210 |
| 8 | 500,160 | 5.3756 | 0.096655 | -0.000595 |
| 9 | 690,240 | 5.5085 | 0.322111 | 0.024422 |
| 10 | **872,640** | **5.6780** | 0.234484 | 0.030307 |
| | | Volatility: | **0.930354** | **0.059634** |
| | | Correlation: | **0.111344** | |

If $\overline{X}$ control charts are not used for controlling the process, $g = 1.8\%$ of the revenue is lost due to scrap, returned parts, and low quality. The monthly cost of using control charts is $K = \$11,000$. The risk-free interest rate is 8% per year. Based on this data, we will provide numerical results for the option valuation techniques discussed in previous sections. Computer programs that we used for option valuation will also be discussed.

## EUROPEAN OPTION MODELS WITH ONE VARIABLE

### *Black-Scholes Formula*

The Black-Scholes formula gives a precise analytic solution for the option. In order to calculate the option value, we simply put the parameter values into Eq. (4).

In our model, the exercise price of the option does not have to be justified at the time intervals following the exercise time. This means that a decision given in one time interval does not affect the succeeding intervals. The manufacturer can change the decision about applying control charts at each time interval. Therefore, the problem can be modeled as a series of European options, with each option expiring at a different time interval.

We will analyze the option of using $\overline{X}$ charts for a one-year time period. We assume that the manufacturer can make a decision to use or not to use the charts each month, implying twelve decision points. At each of the twelve decision points, there is a European option to use the charts. The solution is the sum of all the option values.

Recall that $S$ is the initial extra monthly revenue gained by using control charts. From Eq. (2), this value is \$89,187. As an example, let us calculate the value of the option of using $\overline{X}$ charts at month twelve, using the Black-Scholes formula given in Eq. (4):

$$d_1 = \frac{\ln(\$89{,}187/\$11{,}000) + (0.08 + 0.930354^2/2)(1-0)}{0.930354\sqrt{1-0}} = 2.801 \text{ and}$$

$$d_2 = 2.801 - 0.930354\sqrt{1-0} = 1.871, \text{ so}$$

$$C(S,t) = \$89{,}187 \times N(2.801) - \$11{,}000 \times e^{-0.08(1-0)} N(1.871) = \$79{,}118.$$

By similar calculation, we can find the option value for all months as shown in TABLE 4. The total option value of being able to use or not use the control charts for one year is \$1,022,240.

**TABLE 4**. Option Values Calculated Using the Black-Scholes Formula

| Month | Option value |
|---|---|
| 12 | \$79,118 |
| 11 | \$79,026 |
| 10 | \$78,939 |
| 9 | \$78,854 |
| 8 | \$78,774 |
| 7 | \$78,696 |
| 6 | \$78,622 |
| 5 | \$78,549 |
| 4 | \$78,477 |
| 3 | \$78,405 |
| 2 | \$78,333 |
| 1 | \$78,260 |
| 0 | \$78,187 |
| **Total** | **\$1,022,240** |

*Binomial Lattice*

The underlying variable $S$ can either increase to $uS$, or can decrease to $dS$ in one step of the binomial lattice. The $u$ and $d$ values are calculated using Eq. (5b) and (5c):

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.930354\sqrt{1/12}} = 1.3081$$

and

$$d = 1/u = 0.7645.$$

FIGURE 3 is the twelve step binomial lattice that shows the value of variable $S$ in dollars, where the initial $S$ value is \$89,187 for $u$ = 1.3081, $d$ = 0.7645. In this lattice, moving one step to the right column in the same row means that $S$ has increased to $uS$. Moving one column to the right and one row down means that $S$ has decreased to $dS$.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 89187 | 116665 | 152609 | 199627 | 261130 | 341582 | 446821 | 584484 | 764559 | 1000114 | 1308242 | 1711301 | 2238541 |
| | 68181 | 89187 | 116665 | 152609 | 199627 | 261130 | 341582 | 446821 | 584484 | 764559 | 1000114 | 1308242 |
| | | 52123 | 68181 | 89187 | 116665 | 152609 | 199627 | 261130 | 341582 | 446821 | 584484 | 764559 |
| | | | 39846 | 52123 | 68181 | 89187 | 116665 | 152609 | 199627 | 261130 | 341582 | 446821 |
| | | | | 30461 | 39846 | 52123 | 68181 | 89187 | 116665 | 152609 | 199627 | 261130 |
| | | | | | 23287 | 30461 | 39846 | 52123 | 68181 | 89187 | 116665 | 152609 |
| | | | | | | 17802 | 23287 | 30461 | 39846 | 52123 | 68181 | 89187 |
| | | | | | | | 13609 | 17802 | 23287 | 30461 | 39846 | 52123 |
| | | | | | | | | 10404 | 13609 | 17802 | 23287 | 30461 |
| | | | | | | | | | 7953 | 10404 | 13609 | 17802 |
| | | | | | | | | | | 6080 | 7953 | 10404 |
| | | | | | | | | | | | 4648 | 6080 |
| | | | | | | | | | | | | 3553 |

**FIGURE 3**. The twelve-step binomial lattice.

FIGURE 4 is the binomial lattice that is constructed from the lattice in FIGURE 3, using backward calculation. In order to value the European option, we subtract $K$ = \$11,000 (the monthly cost of using control charts) from the nodes on the last column of the binomial lattice in FIGURE 3. If the result is negative, we put a zero in that node and do not exercise the option in that node.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 79096 | 106457 | 142320 | 189267 | 250701 | 331084 | 436253 | 573844 | 753848 | 989332 | 1297387 | 1700374 | **2227541** |
| | 58061 | 78922 | 106309 | 142180 | 189128 | 250561 | 330943 | 436111 | 573702 | 753705 | 989187 | **1297242** |
| | | 41997 | 57865 | 78764 | 106167 | 142040 | 188987 | 250420 | 330800 | 435967 | 573557 | **753559** |
| | | | 29752 | 41768 | 57692 | 78619 | 106026 | 141898 | 188844 | 250276 | 330655 | **435821** |
| | | | | 20455 | 29475 | 41571 | 57542 | 78477 | 105883 | 141755 | 188700 | **250130** |
| | | | | | 13453 | 20110 | 29237 | 41412 | 57399 | 78333 | 105738 | **141609** |
| | | | | | | 8265 | 13018 | 19806 | 29064 | 41268 | 57254 | **78187** |
| | | | | | | | 4544 | 7721 | 12604 | 19607 | 28919 | **41123** |
| | | | | | | | | 2046 | 3889 | 7129 | 12360 | **19461** |
| | | | | | | | | | 590 | 1333 | 3011 | **6802** |
| | | | | | | | | | | 0 | 0 | 0 |
| | | | | | | | | | | | 0 | 0 |
| | | | | | | | | | | | | 0 |

**FIGURE 4**. Binomial lattice with backward calculations.

Starting from the last column, and using Eq. (5), we go backward one step at a time until we reach time zero. As an example, let us use Eq. (5) to make a one-step backward calculation for the first two elements of the last column in FIGURE 4:

$$\frac{1}{e^{0.08/12}}\left(\frac{e^{0.08/12}-0.7645}{1.3081-0.7645}2{,}227{,}541+\frac{1.3081-e^{0.08/12}}{1.3081-0.7645}1{,}297{,}242\right)=1{,}700{,}374\ .$$

The bold numbers in gray boxes in the last column of FIGURE 4 indicate that it is favorable to apply the control charts at those nodes. The first element in the lattice ($79,096) gives the option value of using the control charts at month twelve.

In order to obtain the option value of using a control chart at month 11, we subtract $K$ = $11,000 from the nodes on column 12 (for period 11) of the binomial lattice in FIGURE 3 and make another binomial lattice using backward calculation until we find the first element. This procedure is repeated for all other months 10, 9,…0. Option values for all months are given in TABLE 5. We see that the option values obtained from binomial lattice are very close to the precise values obtained from Black-Scholes equation (see TABLE 4). Using the binomial lattice approach, we find that the total option value during a one-year time period is $1,022,154. This is comparable to the value of $1,022,240 that was found using the Black-Scholes formula.

**TABLE 5**. Option Values Obtained from Binomial Lattice

| Month | Option value |
|---|---|
| 12 | $79,096 |
| 11 | $79,012 |
| 10 | $78,922 |
| 9 | $78,842 |
| 8 | $78,764 |
| 7 | $78,689 |
| 6 | $78,619 |
| 5 | $78,548 |
| 4 | $78,477 |
| 3 | $78,405 |
| 2 | $78,333 |
| 1 | $78,260 |
| 0 | $78,187 |
| **Total** | **$1,022,154** |

*Monte Carlo Simulation*

FIGURE 5A shows the input window for Monte Carlo Simulation program with the number of sales as a variable and a constant price. FIGURE 5B is the output window. We see that the option value of $1,023,561 is close to the Black-Scholes result of $1,022,240. (See APPENDIX for notes on the program code.)

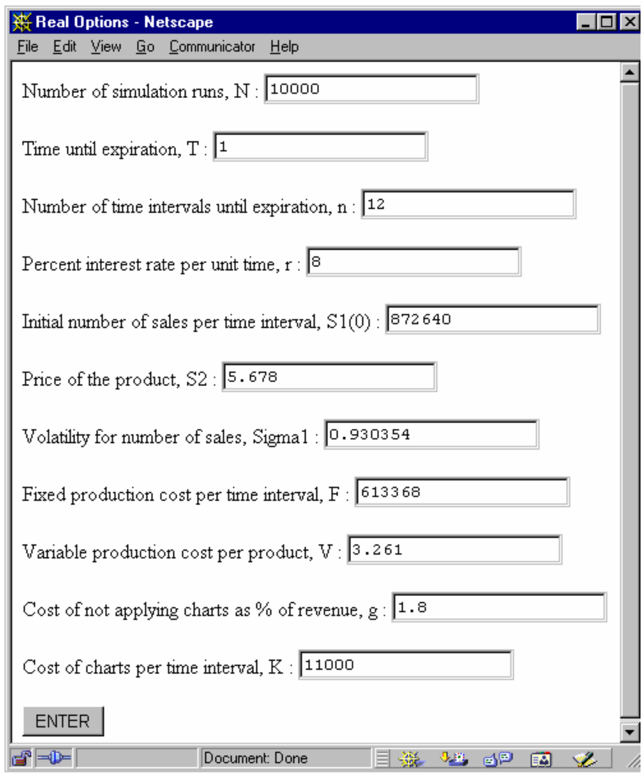


**FIGURE 5A**. Input window of Monte Carlo simulation program.

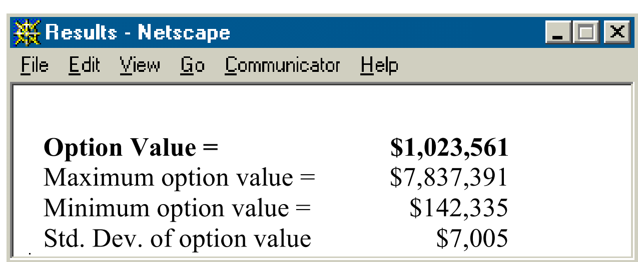


**FIGURE 5B**. Output of Monte Carlo simulation program.

## EUROPEAN OPTION MODELS WITH TWO VARIABLES

### *Pentanomial Lattice*

We use the input values given in FIGURE 6A to evaluate a one-year time period using a pentanomial lattice. The option value is $1,074,054 as shown in FIGURE 6B. This option value is larger than the one obtained using binomial lattice. This happens because there is additional option value due to the variability of the price. (See APPENDIX for notes on the program code.)

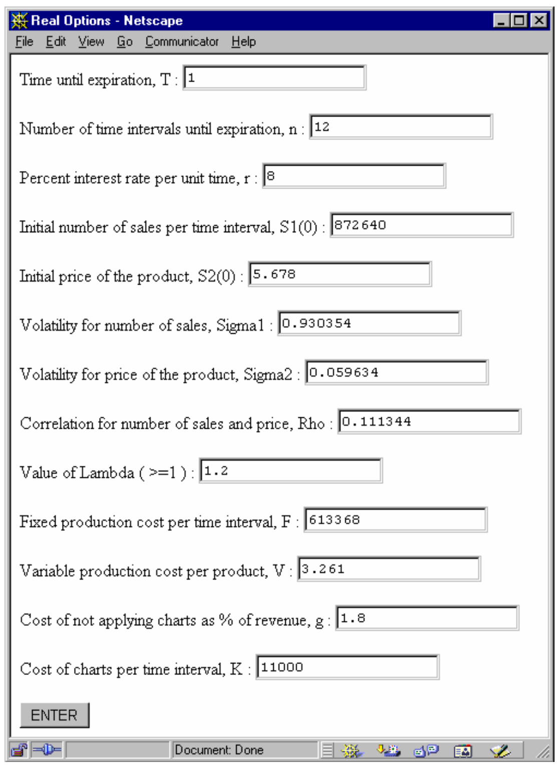


**FIGURE 6A**. Input for pentanomial lattice program.

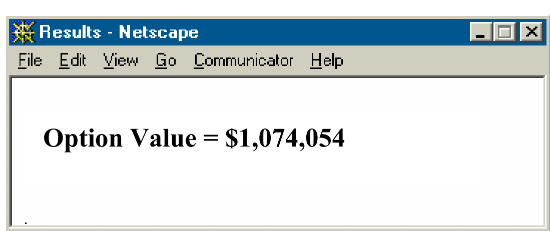


**FIGURE 6B**. Output of pentanomial lattice program.

*Monte Carlo Simulation*

We use the input values given in FIGURE 7A to evaluate a one-year time period using Monte Carlo simulation. FIGURE 7B gives the average option value of $1,076,958 along with the maximum option value, minimum option value, and the standard deviation of the option value, based on 10,000 simulation runs. This simulation average represents a 0.27% difference compared to the pentanomial lattice solution.

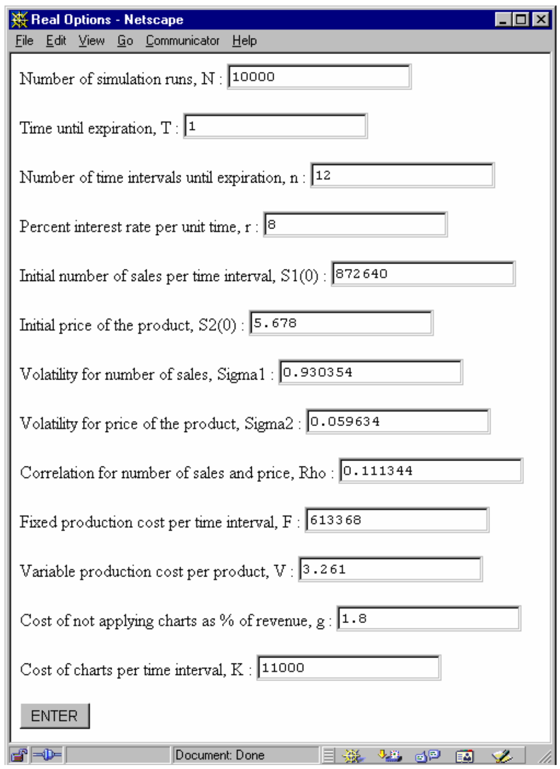


**FIGURE 7A**. Input for Monte Carlo simulation program.

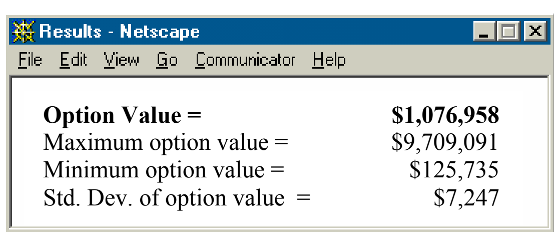


**FIGURE 7B**. Output of Monte Carlo simulation program.

SENSITIVITY OF EUROPEAN OPTION VALUE

Parameters $g$ (cost of not applying charts, which is represented as percent of profit) and $K$ (cost of applying charts per time interval) – are characteristic to the control chart process. FIGURE 8 illustrates how the European option value changes when each of these key factors are independently varied over a range with all other factors held constant.

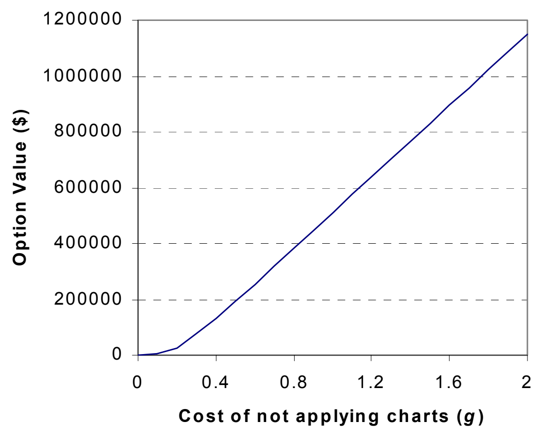


**FIGURE 8A**. $g$ versus European option value (one-variable).

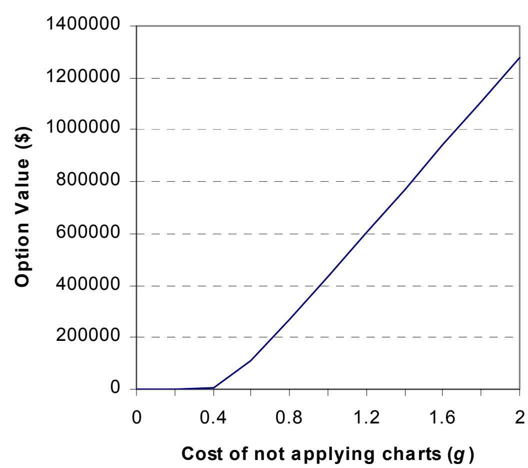


**FIGURE 8B**. $g$ versus European option value (two-variable).

In general, this type of sensitivity analysis may guide the decision maker which ranges suggest that the control charts option should always be used, never used, or used with caution. FIGURES 8A and 8B suggest that we derive more benefit from the option when the cost of not applying control charts is relatively high. FIGURES 8C and 8D suggest that the option is preferable when the control chart cost per time interval is small.

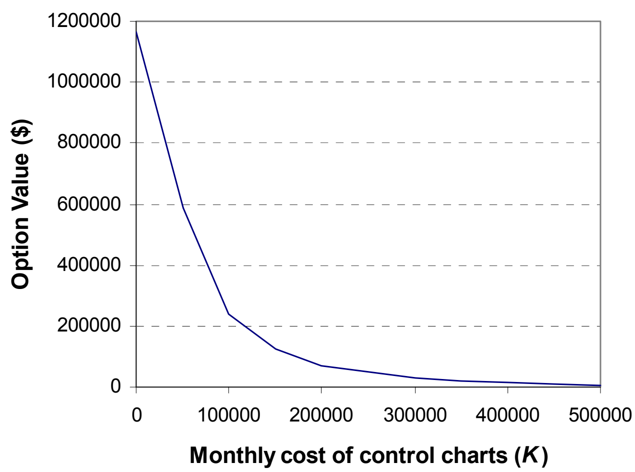


**FIGURE 8C**. *K* versus European option value (one-variable).

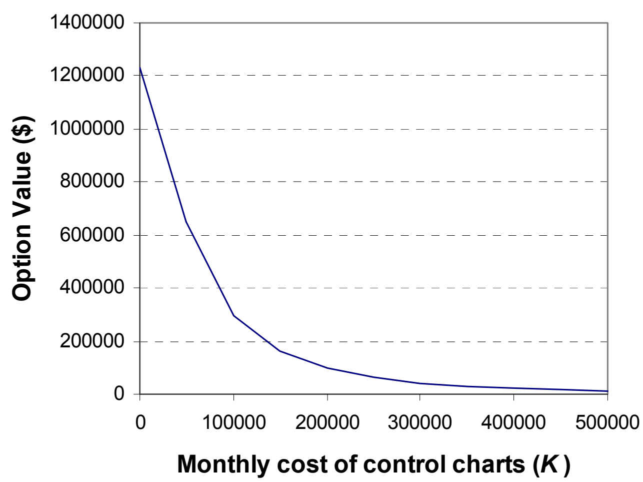


**FIGURE 8D**. *K* versus European option value (two-variable).

## AMERICAN OPTION MODEL WITH ONE VARIABLE

### *Binomial Lattice*

In FIGURE 9A, we assume that control charts are used at all time intervals. In FIGURE 9A, we sum up the elements of the binomial lattice in FIGURE 3 using backward calculations (with *u* and *d* used in European option). The first element in the lattice, \$1,159,435, is the total option value of using control charts in all intervals. However, we did not yet consider the cost of using control charts.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1159435 | 1399983 | 1678698 | 1996266 | 2350171 | 2732659 | 3127750 | 3506903 | 3822795 | 4000456 | 3924725 | 3422603 | 2238541 |
| | 818174 | 981060 | 1166652 | 1373481 | 1597013 | 1827911 | 2049494 | 2234107 | 2337935 | 2293677 | 2000228 | 1308242 |
| | | 573349 | 681812 | 802686 | 933322 | 1068263 | 1197760 | 1305651 | 1366330 | 1340464 | 1168968 | 764559 |
| | | | 398463 | 469103 | 545449 | 624311 | 699991 | 763045 | 798507 | 783390 | 683165 | 446821 |
| | | | | 274152 | 318770 | 364858 | 409087 | 445936 | 466661 | 457827 | 399253 | 261130 |
| | | | | | 186295 | 213229 | 239078 | 260613 | 272725 | 267562 | 233330 | 152609 |
| | | | | | | 124615 | 139721 | 152307 | 159385 | 156368 | 136362 | 89187 |
| | | | | | | | 81655 | 89011 | 93147 | 91384 | 79693 | 52123 |
| | | | | | | | | 52019 | 54437 | 53406 | 46574 | 30461 |
| | | | | | | | | | 31814 | 31212 | 27218 | 17802 |
| | | | | | | | | | | 18241 | 15907 | 10404 |
| | | | | | | | | | | | 9296 | 6080 |
| | | | | | | | | | | | | 3553 |

**FIGURE 9A**. Total option value assuming control charts are used at all intervals.

Using Eq. (11), we find that the exercise price is $K_A$ = \$143,044. The lattice in FIGURE 9B is constructed by subtracting $K_A$ from each element in FIGURE 9A. If the result is negative, we put a zero in that node, which means that we should not exercise the American option at that node.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1016391** | **1256939** | **1535654** | **1853222** | **2207127** | **2589615** | **2984706** | **3363859** | **3679751** | **3857412** | **3781681** | **3279559** | **2095497** |
| | **675130** | **838016** | **1023608** | **1230437** | **1453969** | **1684867** | **1906450** | **2091063** | **2194891** | **2150633** | **1857184** | **1165198** |
| | | **430305** | **538768** | **659642** | **790278** | **925219** | **1054716** | **1162607** | **1223286** | **1197420** | **1025924** | **621515** |
| | | | **255419** | **326059** | **402405** | **481267** | **556947** | **620001** | **655463** | **640346** | **540121** | **303777** |
| | | | | **131108** | **175726** | **221814** | **266043** | **302892** | **323617** | **314783** | **256209** | **118086** |
| | | | | | **43251** | **70185** | **96034** | **117569** | **129681** | **124518** | **90286** | **9565** |
| | | | | | | 1815 | 4100 | **9263** | **16341** | **13324** | 0 | 0 |
| | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 0 | 0 | 0 |
| | | | | | | | | | | | 0 | 0 |
| | | | | | | | | | | | | 0 |

**FIGURE 9B**. Option value after the exercise price is deducted.

The American option value is $1,016,391, which is the first element in the lattice. In this lattice, we assume that once the decision to use charts is exercised, we continue to use them until the expiration date of the option. The bold numbers in gray boxes mean that we should exercise the option at that node. The American option value of $1,016,391 is less than the European option value of $1,022,154, because we do not switch our decision in the American option model, which causes a loss when it is better not to use control charts.

## AMERICAN OPTION MODEL WITH TWO VARIABLES

### *Pentanomial Lattice*

In order to evaluate a one-year time period for the option, we use the input given in FIGURE 10A. The option value is $1,068,299 as shown in FIGURE 10B.

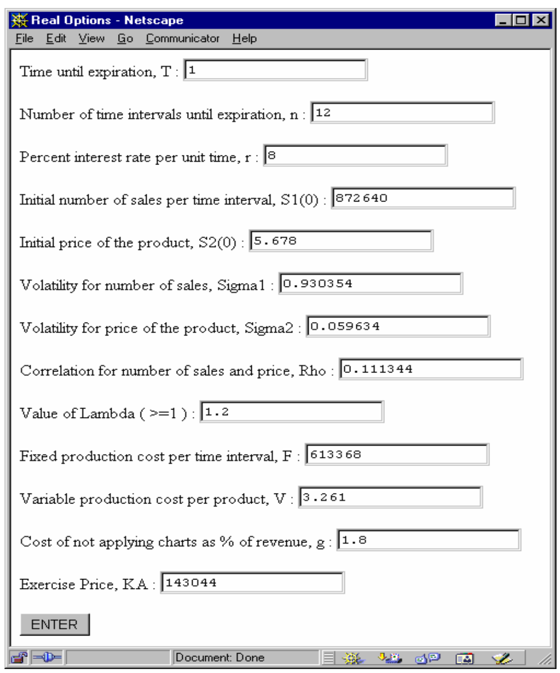


**FIGURE 10A**. Input for the pentanomial lattice program.

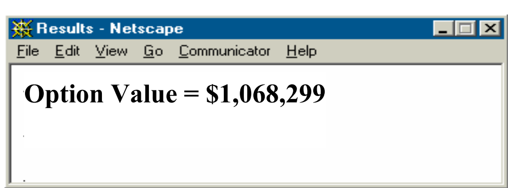


**FIGURE 10B**. Output of the pentanomial lattice program.

SENSITIVITY OF AMERICAN OPTION VALUE

FIGURE 11 illustrates how the American option value changes when *g* and *K* are independently varied over a range with all other factors held constant.

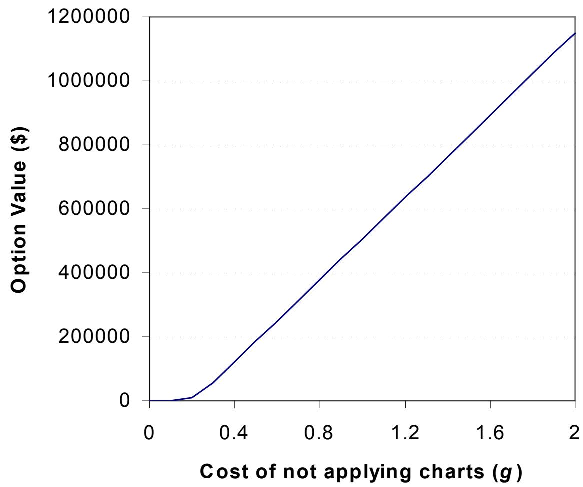


**FIGURE 11A**. *g* versus American option value (one-variable).

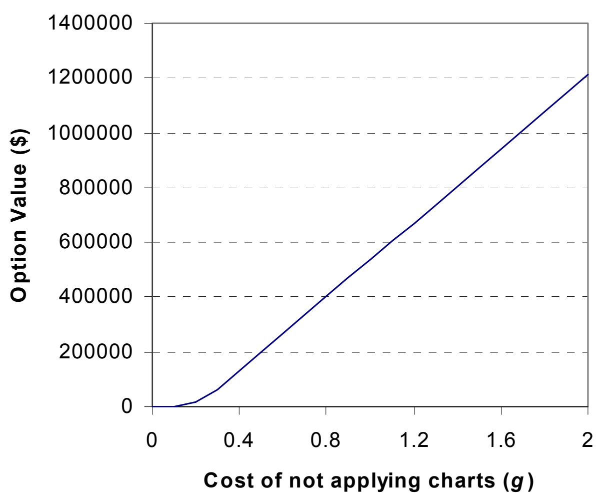


**FIGURE 11B**. *g* versus American option value (two-variable).

FIGURES 11A and 11B suggest that we derive more benefit from the option when the cost of not applying control charts is relatively high. FIGURES 11C and 11D suggest that the option is more valuable when the exercise cost $K$ is small.

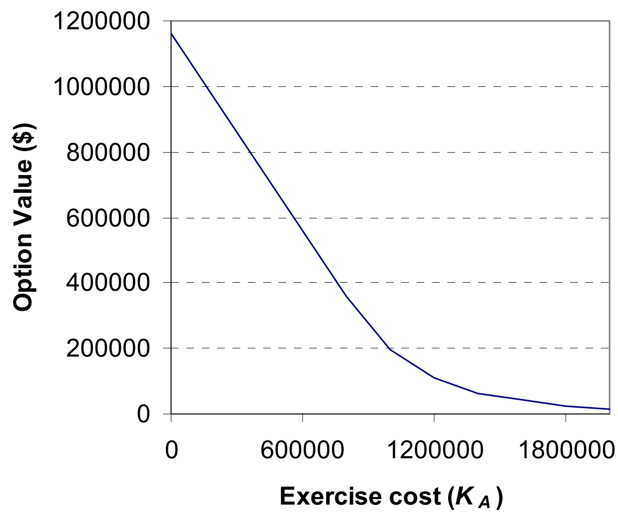


**FIGURE 11C**. $K_A$ versus American option value (one-variable).

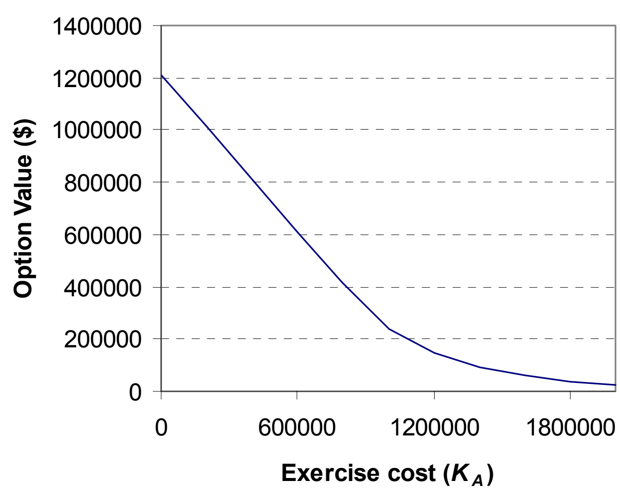


**FIGURE 11D**. $K_A$ versus American option value (two-variable).

## COMPARISON OF OPTION VALUE AND NPV ANALYSIS

TABLE 6 contains the estimated number of sales and price values found by using a linear regression model. The estimated values are then used to find that the net present value (NPV) of the total profit gain by applying control charts over one year is $6,776,863. However, two-variable European option value found by pentanomial lattice for one-year period is $1,074,054.

We see that there is a large difference between option value and NPV results. In this case study, NPV analysis overestimates the value of control charts. Depending on the estimates, NPV analysis may also underestimate the value of a project. Option value analysis protects us from the risk of overestimating or underestimating the value of a project, because it incorporates market dynamics.

**TABLE 6.** Linear Regression Model Estimates

| Month | Number of sales estimate | Price estimate | NPV (profit gain with charts) |
|---|---|---|---|
| 0 | 872,640 | 5.67800 | 78,187 |
| 1 | 1,023,595 | 5.88952 | 96,865 |
| 2 | 1,241,235 | 6.18016 | 125,395 |
| 3 | 1,498,346 | 6.55304 | 162,455 |
| 4 | 1,798,217 | 7.01728 | 210,447 |
| 5 | 2,144,138 | 7.58200 | 272,391 |
| 6 | 2,539,398 | 8.25632 | 352,023 |
| 7 | 2,987,287 | 9.04936 | 453,910 |
| 8 | 3,491,093 | 9.97024 | 583,559 |
| 9 | 4,054,105 | 11.02808 | 747,537 |
| 10 | 4,679,614 | 12.23200 | 953,598 |
| 11 | 5,370,908 | 13.59112 | 1,210,810 |
| 12 | 6,131,277 | 15.11456 | 1,529,685 |
| | | | Total: **6,776,863** |

## CONCLUSIONS

In this paper, we have shown how the option value of employing quality control charts can be determined using a real options framework. The need for this approach was motivated by the inability of existing economic control chart methods to address dynamics in the market conditions with respect to price and number of sales. As these factors change, the benefit of quality control to the organization changes as well.

By connecting the dynamic aspects with the manufacturing operational aspects, we now have a way to address a key issue: the bottom-line profitability associated with the quality control decision. We used an industry case study to demonstrate the application of real options to value control chart decisions.

Although we used $\overline{X}$ control charts in this study, the proposed method can be applied to any type of control chart with proper adjustment of costs. In fact, it can also be applied to other types of decisions that may be affected by market dynamics, such as raw material sourcing or product advertising, if the cost structure is properly modeled.

In summary, the first step is to choose the appropriate option valuation approach as guided by TABLE 1. If there is only one variable that affects the profit, or only one variable is volatile and other variable(s) can be assumed to be stable due to low volatility, then we can use the Black-Scholes formula, binomial lattice, or Monte Carlo simulation. We can use the Black-Scholes formula to get a precise option value; we can use a binomial lattice to see the visual layout of future profit scenarios; or we can use Monte Carlo simulation to see the possible range of profit values. If there are two underlying variables, then possible approaches are pentanomial lattice or Monte Carlo simulation.

There are three possible outcomes of the option value. These outcomes and their interpretations are as follows:

1. If the option value is zero, it means that we do not expect to make more profit by using the control charts until the end of the defined time range.
2. If the option value is positive but we cannot make a larger profit by using control charts during the first time interval, then we do not use control charts at time zero, and we may expect to make more profit by using control charts in the future. The binomial lattice is a good tool that shows when we may expect to use control charts in the future.
3. If the option value is positive and we make a larger profit by using control charts during the first time interval, then we can begin to use the charts at time zero. For American options, we continue to use the control charts until the expiration date of the option. For European options, if the future market conditions are unfavorable for using control charts, then we stop using them until a larger profit is possible again.

TABLE 7 gives the results obtained by the option valuation approaches we considered. All results for our case study are type 3 outcomes, i.e., a positive option value and a larger profit when control charts are used in the first time interval. Therefore, the decision will be to use the control charts at time zero, and continue to use them in all time intervals for an American option. For a European option, the decision will be to use the control charts in all time intervals unless the cost of applying control charts in a time interval is larger than the profit gained by using control charts in that time interval.

**TABLE 7**. A Summary of the Option Valuation Approaches and their Results

| | | European Option Value | American Option Value |
|---|---|---|---|
| **One-variable models** | Black-Scholes Equation | $1,022,240 | |
| | Binomial Lattice | $1,022,154 | $1,016,391 |
| | Monte Carlo Simulation | $1,023,561 | |
| **Two-variable models** | Pentanomial Lattice | $1,074,054 | $1,068,299 |
| | Monte Carlo Simulation | $1,076,958 | |

It may also be promising to investigate the impact of several switching points in the decision to apply the control chart. This will mean that switching costs – an element that is critical in many other types of manufacturing decisions – must be considered. Generalizing these results and implementation issues are also important areas.

## ENDNOTES

[1]This case is based on an actual industry company. However, for confidentiality we have changed identifying information.

## ACKNOWLEDGEMENTS

This work was supported in part by UW-Madison Graduate School Research Grant #10230.

## APPENDIX: NOTES ON PROGRAM CODES

All of the program codes to determine option values were written in Java-Script so that they could be run using Netscape Navigator 2.0 or Microsoft Internet Explorer 3.0 or later versions. They can be obtained by contacting the authors.

In general, the user first enters all parameter values into the input boxes and these values are checked to ensure that they are valid. For the Monte Carlo simulation, the program generates one array for the one variable model and two arrays for the two variable model. The first array contains the $S_1(t)$ values (number of sales for the product) for all time steps until the expiration date of the option, and the second array contains the $S_2(t)$ values (price of the product). Then, value of Eq. (10) at the expiration date is calculated for all simulation runs, and the mean of those values is found. The option value is the present time value of that mean. For the pentanomial lattice models, the program uses arrays to store the lattice values then proceeds with the backward step calculations.

## REFERENCES


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