Individual Observation Process Monitoring Charts for Systems with Response Lags

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Abstract

Previously, it has been held that statistical process control (SPC) and engineering process control (EPC) were two distinct domains for process improvement. However, we design four control policies to integrate the two approaches using a proportional-integral-derivative (PID) feedback controller and Shewhart or moving centerline exponentially weighted moving average (MCEWMA) charts. We specifically consider the impact of these policies for a system with response lags modeled by a second-order dynamic process and autocorrelated nonstationary outside disturbances. These policies are implemented using simulation and applied in a case study of a liquid-tank system. Our investigation shows this approach is effective with respect to overall system performance including quality and cost.

Keywords: quality control, dynamic systems, time series models, simulation

1 Introduction

Many complex systems have a component of *process inertia* causing some type of response lag or transition period between the time a change is made and when the full effect of the change is realized. A car provides an everyday example: when the brakes are applied, it may take several seconds for the car to actually come to a stop. In this paper, we consider a system where the inertia is represented by a second-order dynamic process with an autoregressive integrated moving average (ARIMA) disturbance model. We will return to formal definitions of this system, but for the moment consider its output response shown in Figure 1. In this example, the desired output level is zero for the first ten time periods. Let us assume that although $t = 0$ is when we begin observation, the process has been operating at that level long enough to be in statistical control. At $t = 10$, there is a change in the input to the process because the new desired output level is 20. There is clearly a response lag in the output before the process is able to reach statistical control at a new process level. The main concern is what action, if any, should be taken during the transition period.

Here we develop policies for a second-order system using what we have termed *integrated process monitoring and control* (IPMC). An IPMC policy uses engineering process control (EPC) to make *adjustments* to the process and statistical process control (SPC) to *monitor* the process. A part of the motivation behind our IPMC designs is to use basic tools that are most relevant to practitioners.

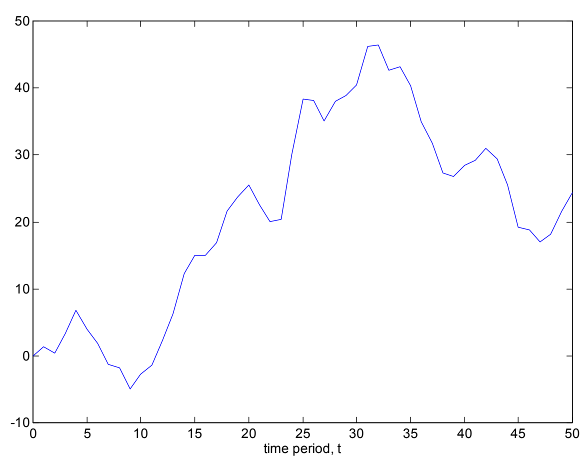

Figure 1. Step Response of A Second-Order System with ARIMA Disturbance

The IPMC designs use a proportional-integral-derivative (PID) controller to provide the EPC component. The PID controller is a robust tool for process adjustments and enjoys a high esteem in the process control literature (e.g., see Seborg, Edgar, and Mellichamp (1989); Ogunnaike and Ray (1994); and Dorf and Bishop (1995)). To provide the SPC component, we compare using both a Shewhart chart and a moving center-line exponentially weighted moving average chart (MCEWMA) for individual observations. The Shewhart chart has been well-discussed in the literature and introductory texts on SPC (see, e.g., Montgomery (1997)). The MCEWMA chart is based on the familiar EWMA chart that is also standard in the literature; however, it adapts the EWMA for the autocorrelated observations given by the ARIMA disturbance model (Montgomery and Mastrangelo (1994)).

We design and evaluate four alternative IPMC strategies as follows:

- Shewhart/PID – simultaneous activation
- MCEWMA/PID – simultaneous activation
- Shewhart/PID – out-of-control activation
- MCEWMA/PID – out-of-control activation

In the first case, the Shewhart monitor and the PID controller are activated simultaneously. In the second case, the MCEWMA monitor and the PID controller are activated simultaneously. In the third case, the Shewhart monitor activates the PID controller when an out-of-control condition occurs. In the fourth case, the MCEWMA monitor activates the PID controller when an out-of-control condition occurs. The first two cases suggest that the results of the monitoring and controlling tools should be considered together which may be done informally in some applications. In the latter two cases, however, the controller is effectively *regulated* by the monitoring mechanism – bringing about a true integration of the two domains.

Previously, it has been held that EPC and SPC were separate domains for process improvement. One of the main reasons for the separation has been that EPC is largely the province of control engineers and SPC that of quality engineers or applied statisticians. Further, each group has had distinct ideas on what constitutes 'improvement'. Traditional SPC has emphasized that any observed difference between a controlled process variable and its desired value may be due to inherent common cause variation (thus no adjustment is necessary to improve the process) or systematic special cause variation (where an adjustment may be necessary to improve the process). The underlying assumption is that there is a disincentive to make an adjustment after every observation period because they are so costly. On the other hand, traditional EPC assumes that the process input variables can be adjusted as frequently as desired, that the adjustments are cost-free, and that the process output variables can be measured without any associated uncertainty (Ogunnaike and Ray (1994)).

Over the years, refinements have been made to this broad picture. MacGregor (1988) points out that the SPC philosophy amounts to a hypothesis test where the null hypothesis is that the process mean is equal to the target (and the alternative hypothesis is that the process mean is not equal to the target). He demonstrates that this hypothesis testing is only appropriate if there is a cost associated with making the adjustment. Furthermore, and perhaps more importantly, the Shewhart approach will give a minimum mean squared error (MMSE) from target only in the case of a steady-state process. That is, the policy of taking action on a process when an out-of-control condition occurs will minimize deviation when the underlying process is a pure gain one. A frequently used steady state process model is $Y_t = KU_{t-1}$, where Y_t is a deviation of the output from target, K is the process gain, and U_t is the deviation of the input from target. This process model means that for any change in the input made at $t-1$, the output will achieve a new steady-state level by the next time period t. However, in the case of first-order process dynamics, where the new level is achieved over several time periods, the SPC approach does not give MMSE control; the EPC approach using a PI controller does.

Several authors have illustrated that the basic SPC tools will also be limited in the case of autocorrelated data. Alwan (1992), Harris and Ross (1991), and Wardell et al. (1994) investigate the robustness of traditional control charts to autocorrelated data that can be represented by an ARMA (1, 1) process. They show that even moderate levels of autocorrelation have a significant impact on a control chart and will give false average run length (ARL) performance. Hunter (1986) suggests that in the case of autocorrelated data, the SPC approach using an EWMA chart can be modified to provide a one-step-ahead prediction of the process output that will provide a discrete dynamic control mechanism analogous to the EPC approach using an Integral controller, assuming that the underlying disturbance follows a white noise process. In the case of stationary stochastic disturbance, specifically an autoregressive moving average (ARMA) process, the EPC approach of applying PI controller can be used to help reduce the overall variation in the output as illustrated by Tsung, Wu, and Nair (1998).

There have been previous attempts to use SPC charts for process control with mixed results. For example, Box and Luceño (1997) use a dyeing process with first-order dynamics and an IMA disturbance to show that the output variance using a Shewhart chart to adjust the process is over two times that of using a PI controller. In this example, the same number of adjustments were made with both the approaches and MMSE was the only objective. However, as pointed out above, the SPC approach should be used when there is a cost for making an adjustment, which implies that adjustments would likely be made less often than every period.

We extend this integration of SPC and EPC to a second-order system with a nonstationary disturbance for two reasons. The first reason is that second-order systems have different responses to control adjustments than do first-order systems. Nembhard and Mastrangelo (1998) show that the capability of a MCEWMA chart to detect a shift is altered on second-order systems perhaps due to the underlying tendency of the output to oscillate under certain conditions. Nembhard (1998) shows that an IPMC design (using Shewhart monitoring and PI control on a stationary process) can be more effective on a second-order system than on a first-order system.

The second reason is that the physical makeup of the system may lend itself to using the more accurate representation of the process. It is known that a second-order system is equivalent to two first-order systems connected in series. In this paper, we look at this type of system based on three liquid tanks. With this specific makeup it may be more undesirable (and perhaps less tractable) to approximate it by two first-order systems, that is by two sets of two tanks, when there are clearly three tanks. Furthermore, representing this system closer to its physical reality may be helpful in understanding systems that are even more complicated such as those composed of networks of components with dynamic behavior.

These types of dynamic systems require us to take a larger view of improvement. Indeed, it is often not a single dimension as may be suggested by one domain or another, but rather a complex tradeoff of multiple decision factors. In our analysis of the performance of IPMC designs, we consider the traditional output variation and number of adjustments as well as the magnitude of adjustments and the number of observations outside monitoring limits (alarms). We show that this combination of monitoring and adjustment is an effective approach for second-order systems during the transition period with respect to overall system performance.

The remainder of the paper is organized as follows. Section 2 discusses the second-order system and its state-space representation. Section 3 shows how to represent the ARIMA disturbances using the same state-space form. Section 4 gives the equations that represent the EPC and SPC components of the IMPC design. Section 5 shows a graphical simulation model of the second-order system with disturbance and also the integrated control mechanism applied to the system. In Section 6, we present a case study of a liquid-tank system and analyze the results. Finally, in Section 7 we summarize the work and make some concluding remarks.

2 Second-Order Dynamics in the Startup Problem

A second-order system arises physically from two first-order systems connected in series. For example, Figure 2 shows a liquid tank system adapted from Box, Jenkins, and Reinsel (1994). At time t, U_t is the volume of liquid in Tank 1, W_t is the volume of liquid in Tank 2, and Y_t is the volume of liquid in Tank 3. The tanks are connected through pipe P and mechanical devices in the system allow us to change the volume U_t in Tank 1 according to any pattern without regard to the level in Tanks 2 and 3. This system could be viewed as two first-order systems by considering one system as Tanks 1 and 2 where the input is U_t and the output is W_t and considering a second system as Tanks 2 and 3 where the input is W_t and the output is Y_t. By considering the entire three-tank system, the input is U_t and the output is Y_t.

The steady-state operation of this type of system has been well-investigated. Consider that the Shewhart philosophy is that the process is monitored for a sign of a special cause, e.g., a leak in the pipes. The underlying assumption is that the process will operate at a given level unless an unexpected event occurs. By contrast, we focus on the startup problem where a process is operating at one level for some-time (and has presumably reached a steady-state condition) and then has to achieve a new operating level of statistical control. A specific application within the framework of the given example is when the tanks are emptied for maintenance and have to be refilled with the same product. Another application is when they are emptied and refilled to accommodate a new product as in the case of a chemical manufacturer that alternately uses the tanks for two different products.

We consider the standard Laplace transfer function form and the state-space form of the second-order system, which are both commonly used in control engineering. The state space form will be used to facilitate the simulation modeling in Section 5. Our specific interest in this system is what happens to the output Y in the time periods immediately following a step-change in the input U. The step input is given by

$$U_t = \begin{cases} 0 & \text{if } t < c; \\ M & \text{otherwise,} \end{cases} \tag{1}$$

where c is the time at which the step change of magnitude M occurs. The standard Laplace transfer function form of the second-order process is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau^2 s^2 + 2\xi\tau s + 1} \tag{2}$$

where *Y*(*s*) is the output of the system, *U*(*s*) is the input of the system, *K* is the steady-state process gain, τ is the process time constant (or equivalently, the speed of the response or response time), and ξ is a measure of the degree of oscillation in the process response (damping) after a perturbation (e.g., see Ogata (1987, 1990) or Seborg et al. (1989)). In general, a process of order *n* can be represented by the transfer function

$$\frac{Y(s)}{U(s)} = \frac{b_o s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \tag{3}$$

where the *a*'s and *b*'s are appropriate coefficients (e.g., see Ogata (1987)).

A continuous state-space representation of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

of the Equation (3) transfer function is given by the following two sets of equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u \tag{4}$$

$$y = \begin{bmatrix} (b_1 - a_1 b_0) & (b_2 - a_2 b_0) & \cdots & (b_n - a_n b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_0 \end{bmatrix} u. \tag{5}$$

Note that using Equation (1) with *c* = 10, $M = 1$ and Equation (2) with $\tau^2 = 15$, $K = 20$, and $\xi = 0.6$ gives the second-order response curve shown in Figure 3.

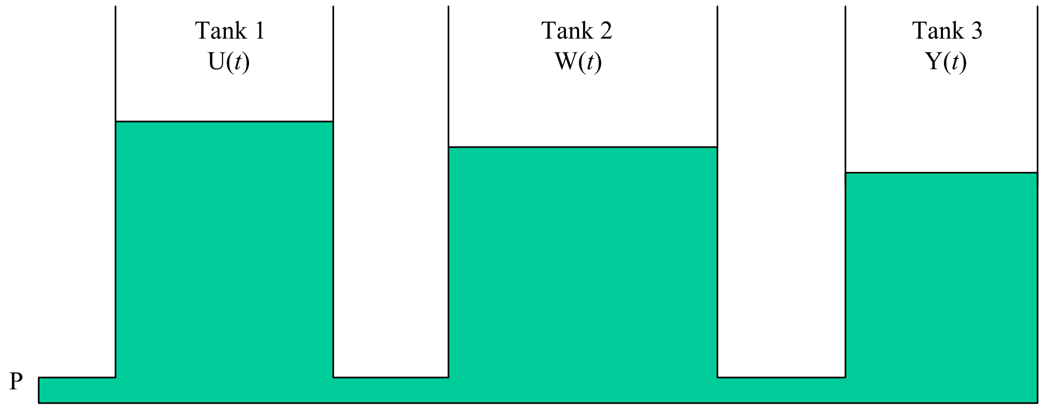

Figure 2. A Pipe Connects Three Liquid Tanks to Make a Second-Order Dynamic System

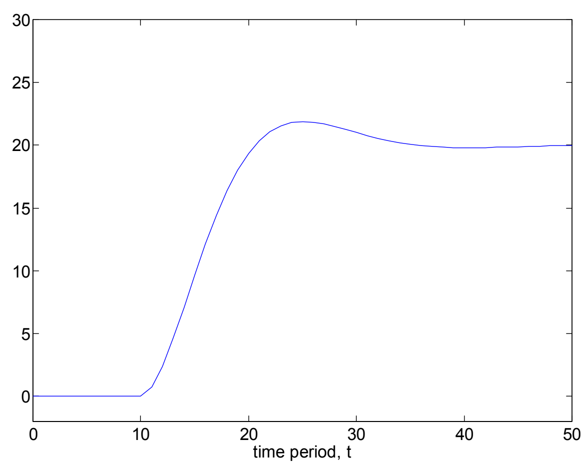

Figure 3. A Second-Order Response to a Step Change in Input

3 Representing Disturbances Using the ARIMA Process

As discussed in Section 1, many EPC policies for MMSE assume that the output response, Y_t is noise-free. In practice, however, Y_t typically has an associated disturbance. This disturbance is represented here using a nonstationary stochastic ARIMA process. The ARIMA process is a widely used mathematical model that is a special case of the linear filter of white noise. As with the second-order output response, the ARIMA model can be represented in several forms. Here we will consider both the rational polynomial form which is often used in quality engineering and the state-space form which will be used in the simulation modeling in Section 5.

The ARIMA process of order (*p*, *d*, *q*) can be represented as

$$\Phi_p(B)\nabla^d y_k = \Theta_q(B)u_k \tag{6}$$

where *B* is the backward shift operator defined by $B^m z_k = z_{k-m}$, ∇ is the difference operator defined by $\nabla = 1 - B$, Φ_p is the autoregressive operator defined by $\Phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$, and Θ_q is the moving average operator defined by $\Theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$ (Box et al. 1994).

A discrete state-space representation of the form

$$x(k+1) = \overline{A}x(k) + \overline{B}u(k)$$

$$y(k) = \overline{C}x(k) + \overline{D}u(k)$$

for the Equation (6) ARIMA(p, d, q) process is given by the following two equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ \vdots \\ x_r(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \phi_r & \phi_{r-1} & \cdots & \cdots & \phi_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_r(k) \end{bmatrix} + \begin{bmatrix} 1 \\ \psi_1 \\ \vdots \\ \vdots \\ \psi_{r-1} \end{bmatrix} u(k) \tag{7}$$

$$y(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_r(k) \end{bmatrix} + \begin{bmatrix} N \end{bmatrix} u(k) \quad (8)$$

where $r = \max(p + d, q + 1)$ and N is 0 or 1 depending on whether the process is subject to an additional white noise term (in our examples, we assume $N = 0$). The ψ weights can be found from the ϕ and θ weights by equating coefficients of like powers in the expansion

$$(\psi_0 + \psi_1 B + \cdots)(1 - \phi_1 B - \cdots - \phi_p B^p)(1 - B)^d = 1 - \theta_1 B - \cdots - \theta_q B^q .$$

Equation (7) is known as a state or transition equation and Equation (8) is known as an observation equation. Figure 1 shows a second-order output response with an ARIMA (1,1,1) disturbance.

4 Monitoring and Control of the Startup Process

During a process or system transition, a key issue is making process adjustments in the startup phase. In integrating EPC and SPC to make adjustments in this situation, we compare the ability to activate both the monitoring and control simultaneously with the ability to use the monitoring to regulate the controller to start and stop adjustments to the input. The IPMC mechanisms studied here all combine a PID controller with a MCEWMA or Shewhart control chart for the case where output observations during the transition period of a second-order process are autocorrelated.

An important question is: What represents and improvement in control? One aspect is that the benefit of the combination stems from reducing the mean squared error. By integrating basic EPC and SPC policies, it is possible to reduce the output variance in some systems beyond the capabilities of either tool individually (Vander Wiel et al. (1992) and MacGregor (1988)). Montgomery et al. (1994) demonstrate this using a modified funnel experiment (as used by MacGregor (1990)) in combination with an SPC chart monitoring deviation from target. Another aspect is that the benefit is associated with having to make an adjustment less often. This may be particularly relevant for systems where there is a distinct cost associated with making each adjustment or where an operator must manually impose the adjustment.

EPC Policies

PID controllers are standard in EPC. Several texts cover the basics of PID controller design (see e.g., Seborg et al. (1989) or Dorf and Bishop (1995)). Although, these tools are common for making adjustments to a process, they are usually designed for steady-state behavior and do not perform effectively during the startup. Manual adjustments are often used for this reason.

A typical control objective is MMSE control, or to minimize the variance of the output deviations from a target or setpoint. One criteria in developing an adjustment policy under this objective is cost. If the cost of a process adjustment is zero, then an adjustment is made at every sampling interval. If the cost of adjustment is nonzero, then an adjustment is made only if a substantial deviation occurs. There are many optimal adjustment policies for key underlying process representations, disturbance models, and adjustment costs for MMSE control. For example, integral control is an optimal policy for a steady-state process with nonstationary disturbance and zero adjustment costs (Box et al. (1974) and Box and Luceño (1997)). For the same process and disturbance models with nonzero adjustment costs, the Shewhart control chart and the EWMA chart (with appropriately chosen parameters) are equivalent optimal adjustment policies in terms of minimizing the output variance (MacGregor (1988)).

For the case of a change in the process mean, Hunter (1986) uses an empirical equation to arrive at MMSE control:

$$\hat{y}_{t+1} = \hat{y}_t + \lambda_1 e_t + \lambda_2 \sum e_t + \lambda_3 \nabla e_t \qquad (9)$$

where $\hat{y}_{t+1}$ is the predicted value at time $t+1$; $\hat{y}_t$ is the predicted value at time t; λ_1, λ_2, and λ_3 weight the historical data; e_t is the observed error from target at time t ; and ∇ is the first-difference operator. Following Box and Luceño (1997), the values of λ_1, λ_2, and λ_3 are chosen to minimize the sum of the squared error. This constitutes the PID control equation with one quantity that is proportional to e_t, a second that is a function of the sum (integral) of e_t, and a third that is a function of the difference (derivative) e_t. We will apply Equation (9) in the IPMC models discussed in the Section 5.

SPC Policies

In SPC, the Shewhart chart has enjoyed widespread industry use. Because it is a robust procedure, we consider its performance in the startup phase. We specifically consider two individual observation process monitoring charts. The first type of chart is the Individual Values and Moving Range chart. This chart was used in this investigation because of its sensitivity for periodic measurements from a continuous process. However, during a transition, observations are likely to be autocorrelated (e.g., see Box, Jenkins, and Reinsel (1994); and Montgomery (1995)). Therefore, the second type of chart we consider is a MCEWMA chart. In the simulation model discussed in Section 5, we represent the monitoring limits for these charts as discussed below.

The standard Individual Values and Moving Range chart is specified by two sets of limits. The three-sigma limits about the mean response are given by:

$$\begin{aligned} \text{UCL} &= \overline{R}_j + 3/d_2\,\overline{mr} \\ \text{CL} &= \overline{R}_j \\ \text{LCL} &= \overline{R}_j - 3/d_2\,\overline{mr} \end{aligned} \tag{10}$$

where the moving range, *mr*, is based on the successive differences between the individual values of R_j and hence a values of d_2 = 1.128 is used for subgroups of size 2. (These formulae are often written with X instead of R_j. The current notation was chosen to emphasize the charting of the response variable.) In practice, when the process mean is changing to a new level, we can use the $\overline{R}_j$ of a prior realization at the new process level – for example, data from the transition made last month.

The three-sigma limits about the estimate of the variance are given by:

$$\begin{aligned} \text{UCL} &= D_4\,\overline{mr} \\ \text{CL} &= \overline{mr} \\ \text{LCL} &= 0 \end{aligned} \tag{11}$$

where a value of D_4 = 3.268 is used for subgroups of size 2.

Montgomery and Mastrangelo (1991) introduce a control chart methodology for application to autocorrelated data. The MCEWMA is a procedure that approximates the true underlying time series model with the familiar EWMA. The EWMA is typically used with independent data for either subgroup or individual measurements, but can be applied to correlated data if some modifications are made to the procedure. The EWMA for independent data is defined as

$$Z_t = \lambda Y_t + (1-\lambda) Z_{t-1}.$$

By replacing λ with $1-\theta$ and rearranging, the EWMA is equivalent to the IMA (1,1) model (Box et al. (1994)). The EWMA with $\lambda = 1-\theta$ is the optimal one-step-ahead forecast for the IMA (1,1) process.

The MCEWMA uses Z_t as the center-line for period $t+1$. The upper and lower limits are given by a three sigma range:

$$\begin{aligned} \text{UCL} &= Z_t + 3\sigma_t \\ \text{CL} &= Z_t \\ \text{LCL} &= Z_t - 3\sigma_t \end{aligned} \tag{12}$$

where σ_t is the standard deviation of the single-period forecast errors. The value of σ_t can be estimated by several methods. We use the smoothed variance approach:

$$\hat{\sigma}_t^2 = \alpha e_t^2 + (1-\alpha)\hat{\sigma}_{t-1}^2,$$

where e_t is the one step-ahead prediction error and $\alpha = 0.05$. Note that in using the smoothed variance method, the estimate of σ_t is updated each period, which in turn, results in the varying control limits. In essence, this approach bases the estimate of σ_t on recent process history. The initial values for Z_t and $\hat{\sigma}_t^2$ are the mean and the variance of the noise model, respectively. To test for statistical control, the output observation is compared to these limits.

5 Simulation Model of the Liquid-Tank Startup System

We developed a simulation model using SIMULINK (1997) to represent the EPC/SPC policies executed on the liquid-tank second-order dynamic system with ARIMA disturbance. A schematic of the model is shown in Figure 4. In this section, we will discuss the operation and block components of this model.

Selecting any of the simulation model blocks reveals an input environment for the model parameters. Primarily, the state-space representation given by the A, B, C, and D matrices of the ARIMA filter (Equations (4) and (5)) and the second-order process (Equations (7) and (8)) must be provided as input.

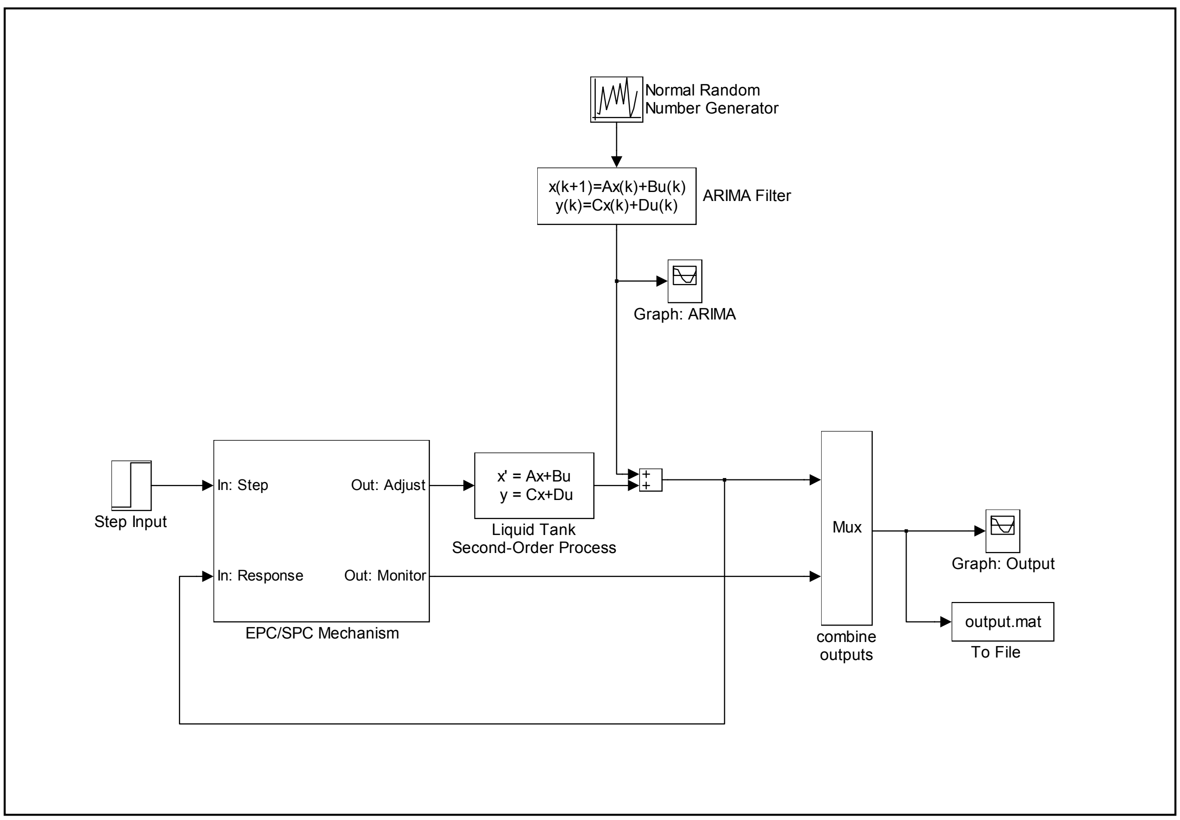

Figure 4. The SIMULINK model of the Liquid-Tank Second-Order Dynamic Process with ARIMA Disturbance

The NORMAL RANDOM NUMBER GENERATOR block provides random shocks from a normal distribution with any value for the mean and variance. Replications of the simulation are achieved by specifying a new random number stream in this block. The ARIMA FILTER block provides the discrete state-space form of the linear filter process. The ARIMA process is plotted during the simulation run via the GRAPH: ARIMA block.

The STEP INPUT block provides a shift change in the input at a specified time which is given by Equation (1). The EPC/SPC MECHANISM contains the EPC controller given by Equation (9) and the SPC monitoring limits given by either Equation (10) or Equation (12) as the case requires. The SECOND-ORDER PROCESS block represents the dynamics of the liquid-tank system. The noise process and the plant process are summed together to model the process. The COMBINE OUTPUT block creates one vector from the vectors of the summed noise process, plant process and monitoring limits so that they can be plotted on the same graph via the GRAPH: OUTPUT block. This combined output vector is saved using the TO FILE block so that the results can be further analyzed and compared with other runs. We have used this basic model in early studies on the performance of dynamic systems involving stationary disturbances (Nembhard (1998) and Nembhard and Mastrangelo (1998)). We have also discussed some of the particulars of building SIMULINK models in Nembhard and Nembhard (1996).

The critical component of the present work is designing new EPC/SPC monitoring mechanisms and testing their performance. In the next section, we will investigate four cases; each one requires a different mechanism.

6 Case Study

In this investigation, we consider four IPMC strategies for a system consisting of a second-order dynamic process and an ARIMA disturbance process. These cases are summarized as follows: (1) the PID controller and the Shewhart monitor are activated simultaneously; (2) the PID controller and the MCEWMA monitor are activated simultaneously; (3) the Shewhart monitor activates the PID controller when an out-of-control condition occurs; and (4) the MCEWMA monitor activates the PID controller when an out-of-control condition occurs.

We employ a model of the liquid-tank system that can be represented (using Equation (2)) by a second-order dynamic process:

$$\frac{Y(s)}{U(s)} = \frac{20}{15s^2 + 2(0.6)(\sqrt{15})s + 1}.$$

By Equations (4) and (5), the matrices for the state-space representation of this process are

$$A = \begin{bmatrix} -0.309 & -0.067 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 1.333 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix} \tag{13}$$

In each case there is a unit step change given by Equation (1) at time 10. With this step, the new target level that the process must reach is 20. An example of the undisturbed response is as shown in Figure 3.

The ARIMA (1, 1, 1) process representing the disturbance to the system is given by

$$y_k = 1.9 y_{k-1} - 0.9 y_{k-2} + u_k + 0.6 u_{k-1}$$

with $\sigma_u^2 = 9$. By Equations (7) and (8), the matrices for the state-space representation of the disturbance are

$$\overline{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0.9 \end{bmatrix} \quad \overline{B} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$
$$\overline{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \overline{D} = \begin{bmatrix} 0 \end{bmatrix} \tag{14}$$

One sample path generated by this ARIMA model is shown in Figure 5.

The state-space matrices given by Equations (13) and (14) are then used in the simulation model discussed in the previous section.

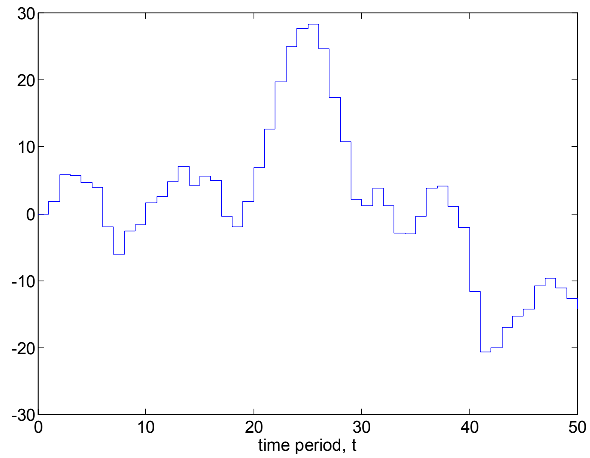

Figure 5. Sample path of an ARIMA (1,1,1) model with $\theta_1 = -0.6,\ \phi_1 = 0.9,$ and $\sigma^2 = 9$.

Case 1: Shewhart/PID – Simultaneous Activation

In Case 1, the IMPC Mechanism has a Shewhart monitor and a PID controller. The monitoring begins as soon as the PID controller is applied to the process. The Shewhart monitoring serves to evaluate the effectiveness of the PID controller (via the process output). An adjustment is made in each period so the monitoring mechanism does not interfere with the capacity of the PID controller to make adjustments. Table 1 shows a typical series of output observations and adjustments using this design. The Shewhart upper, center, and lower monitoring limits for the new level are 29, 20, and 11, receptively. After the step change, two observations are outside the control limits (indicated by *). These data are shown graphically in Figure 6 and represent the sample path for one simulation run under this policy.

TIME	OUTPUT	ADJUSTMENT		TIME	OUTPUT	ADJUSTMENT
0	0	0		26	24.4289	-5.1053
1	3.4949	-6.9897		27	22.7140	-3.4123
2	4.9101	-8.9844		28	21.4223	-3.0612
3	0.4952	-1.2181		29	20.2770	-1.7199
4	-2.7031	2.3928		30	14.1784	10.2754
5	-4.9071	6.4993		31	15.0774	7.5126
6	1.7422	-5.5851		32	24.5885	-8.4966
7	5.3628	-9.0493		33	28.5656	-12.2947
8	6.0682	-10.6056		34	15.3228	12.6241
9	2.1220	-4.9816		35	8.6157*	19.1486

10	-2.0839	42.9913		36	15.8780	5.5586
11	7.3745	15.8695		37	25.0676	-6.0688
12	19.2922	4.5335		38	25.4052	-3.6975
13	31.1779*	-12.4891		39	21.0966	1.7034
14	28.7569	-7.0464		40	23.5643	-6.9838
15	20.4215	2.5010		41	25.4793	-10.3974
16	14.9560	7.3971		42	20.9747	-1.9272
17	14.4253	7.7476		43	13.1627	10.9510
18	14.3796	10.5377		44	12.8105	10.2924
19	14.5807	12.4868		45	19.6590	0.1247
20	23.4230	-3.4179		46	22.3031	-0.5504
21	28.1801	-9.5750		47	20.5444	2.9814
22	21.1692	3.0143		48	21.1077	-0.2608
23	13.8051	12.0307		49	26.0814	-10.5868
24	18.2715	1.1518		50	16.7265	8.8038
25	23.6635	-5.4851				

Table 1. A Typical Series of Output Observations and Adjustments Made in Case 1.
* Observation outside monitoring limits

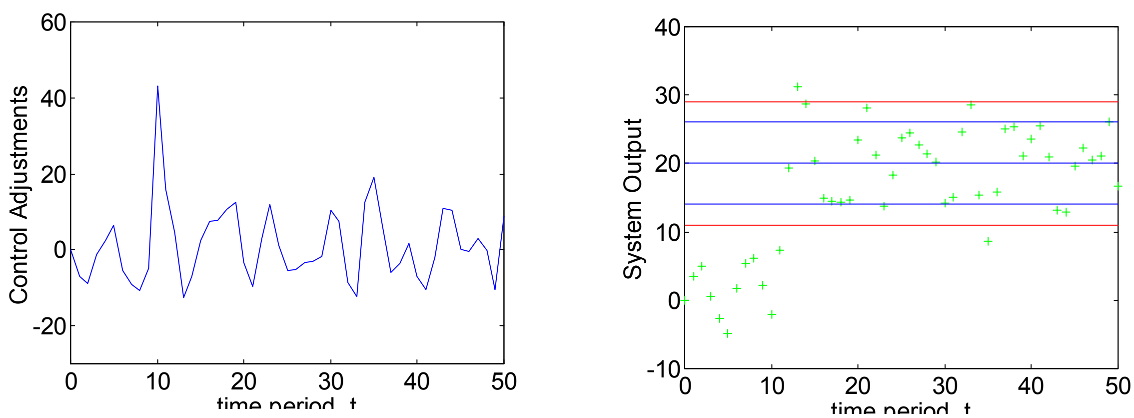

Figure 6. The IPMC Mechanism is Designed to Activate the PID Controller and Shewhart Monitor Simultaneously at $t = 0$. An Adjustment is Made Each Period.

Case 2: MCEWMA/PID – Simultaneous Activation

In Case 2, the IMPC Mechanism has a MCEWMA monitor and a PID controller. As in Case 1, the monitoring begins as soon as the PID controller is applied to the process. The MCEWMA monitoring serves to evaluate the effectiveness of the PID controller (via the process output) but does not interfere with its adjustment. Table 2 shows a typical series of output observations, monitoring limits, and adjustments using this design. Figure 7 shows the sample path for one simulation run under this policy. An adjustment is made in each period. After the step change at $t = 10$, three observations are outside the monitoring limits.

TIME	OBSERVATION	UCL	CL	LCL	ADJUSTMENT
0	0	0	0	0	0
1	-0.8376	7.2877	0	-7.2877	1.6753
2	-4.9579	7.1245	-0.1675	-7.4596	9.7155
3	-0.0360	6.2708	-1.1256	-8.5220	-0.7778
4	6.6835	6.4589	-0.9077	-8.2743	-11.1082
5	2.7166	8.2859	0.6106	-7.0648	-1.9710
6	-2.8943	8.6947	1.0318	-6.6312	5.1174
7	-4.7756	7.9615	0.2465	-7.4684	6.6380
8	-3.4540	7.0649	-0.7579	-8.5807	5.3178
9	1.2286	6.5284	-1.2971	-9.1226	-1.5679
10	5.5882	7.0310	-0.7920	-8.6150	34.5231
11	24.5815	8.4997	0.4841	-7.5316	-10.7325
12	34.8197*	16.0679	5.3035	-5.4608	-20.8779
13	27.2558*	25.1035	11.2068	-2.6900	-6.6294
14	17.6730*	29.0580	14.4166	-0.2248	3.5723
15	12.9769	29.6686	15.0679	0.4671	8.3556
16	15.6626	29.1907	14.6497	0.1086	4.2814
17	18.5099	29.3236	14.8522	0.3809	3.0411
18	18.2223	30.0244	15.5838	1.1432	6.0514
19	19.0547	30.5015	16.1115	1.7215	4.3397
20	24.3326	31.0452	16.7001	2.3551	-5.9094
21	28.4273	32.6823	18.2266	3.7710	-13.0149
22	24.3123	34.9719	20.2668	5.5616	-6.0512
23	16.9460	35.7575	21.0759	6.3942	4.2547
24	11.8955	34.9104	20.2499	5.5894	11.6151
25	9.9614	33.3798	18.5790	3.7782	16.6322
26	17.8171	31.8073	16.8555	1.9037	4.3542
27	25.8292	31.9275	17.0478	2.1682	-6.0708
28	26.9733	33.8417	18.8041	3.7665	-6.7935
29	17.1460	35.5996	20.4379	5.2763	9.4552
30	8.5585	34.8975	19.7796	4.6616	21.0033
31	17.4769	32.9496	17.5353	2.1211	2.6952
32	30.0130	32.8606	17.5237	2.1867	-15.3087
33	31.7426	35.7349	20.0215	4.3082	-15.5242
34	24.8633	38.3908	22.3657	6.3406	-6.5330
35	19.3404	38.8276	22.8653	6.9029	-1.9040
36	17.0948	38.0778	22.1603	6.2427	0.5522
37	20.3146	37.0577	21.1472	5.2367	-4.5868
38	18.8445	36.8134	20.9807	5.1480	1.3921
39	19.6497	36.3198	20.5534	4.7871	-0.2297
40	19.4059	36.0624	20.3727	4.6830	1.1239
41	20.7410	35.7931	20.1793	4.5656	-1.3547
42	14.1288	35.8281	20.2917	4.7553	12.4061
43	7.0809	34.6278	19.0591	3.4904	24.5719
44	15.9539	32.5654	16.6635	0.7615	7.4830
45	27.4731	32.3452	16.5215	0.6979	-8.6177
46	25.0800	34.7954	18.7119	2.6283	-0.8754
47	14.9834	36.1020	19.9855	3.8690	13.9996
48	9.1049	35.0909	18.9851	2.8793	20.6236
49	18.5160	33.3059	17.0090	0.7121	2.6743
50	23.6417	33.5319	17.3104	1.0889	-0.9889

Table 2. A Typical Series of Output Observations, Monitoring Limits, and Adjustments Made in Case 2.
* Observation outside monitoring limits

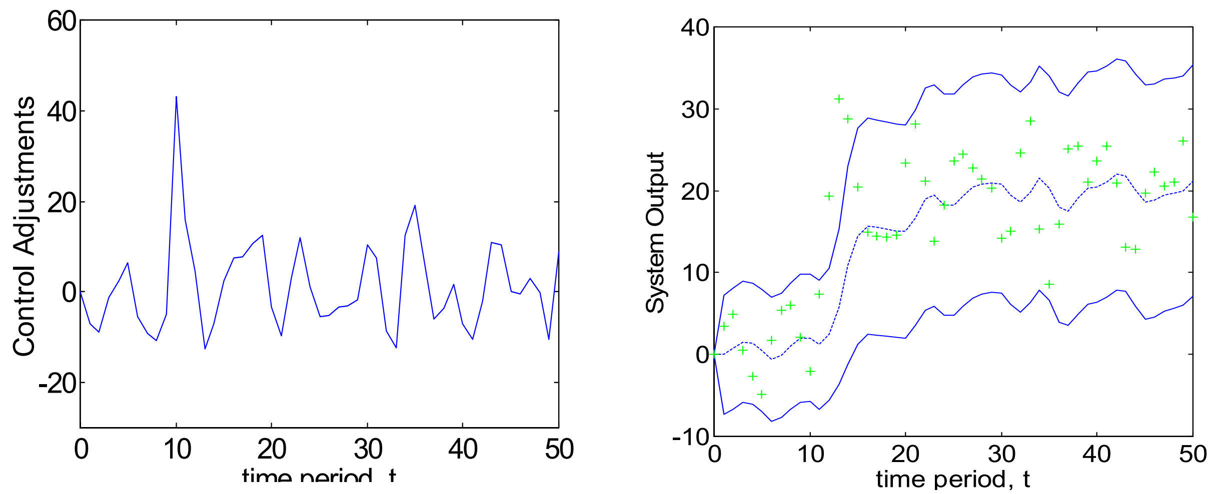

Figure 7. The IPC Mechanism is Designed to Activate the PID Controller and MCEWMA Monitor Simultaneously at $t = 0$. An Adjustment is Made Each Period.

Case 3: Shewhart/PID – Out-of-Control Activation

In Case 3, the IMPC Mechanism has a Shewhart monitor and a PID controller which is the combination used in Case 1. The distinction, however, is that the MCEWMA monitoring serves to regulate the PID controller because it allows adjustments only when the process output exceeds the control limits. Table 3 shows a typical series of output observations and adjustments using this design. Figure 8 shows the sample path for one simulation run under this policy. After the step change at $t = 10$, 14 observations are outside the control limits.

TIME	OBSERVATION	ADJUSTMENT		TIME	OBSERVATION	ADJUSTMENT
0	0	0		26	13.8092	0
1	1.7961	-3.5922		27	8.0069*	13.5552
2	4.6800	-8.9305		28	7.7837*	17.5260
3	0.6427	-0.8866		29	8.3668	23.1432
4	-3.7397	5.1583		30	18.3472	0
5	-3.5078	3.8769		31	27.6624	0
6	-6.1074	11.1968		32	28.6069	0
7	-4.4726	8.8020		33	23.8407	0
8	5.9721	-9.5983		34	22.8622	0
9	6.5541	-7.2391		35	24.5748	0
10	1.3364*	43.6764		36	27.7155	0
11	10.3318*	18.3917		37	27.1136	0
12	32.4138*	-19.3311		38	23.1685	0
13	32.8264*	-9.7389		39	19.2862	0
14	23.4404	0		40	9.0909*	-5.4291
15	21.5511	0		41	-4.9876*	25.3375

16	18.7361	0		42	3.3507*	17.8164
17	12.1901	0		43	25.0292	0
18	10.0481*	14.0542		44	33.8300*	-16.0679
19	19.2895	0		45	27.5405	0
20	29.9062*	-13.0228		46	21.1976	0
21	33.4183*	-19.2846		47	14.5087	0
22	28.6645	0		48	9.2203*	20.5702
23	23.7438	0		49	15.5385	0
24	20.3164	0		50	24.1447	0
25	17.9866	0				

Table 3. A Typical Series of Output Observations and Adjustments Made in Case 3.
* Observation outside monitoring limits

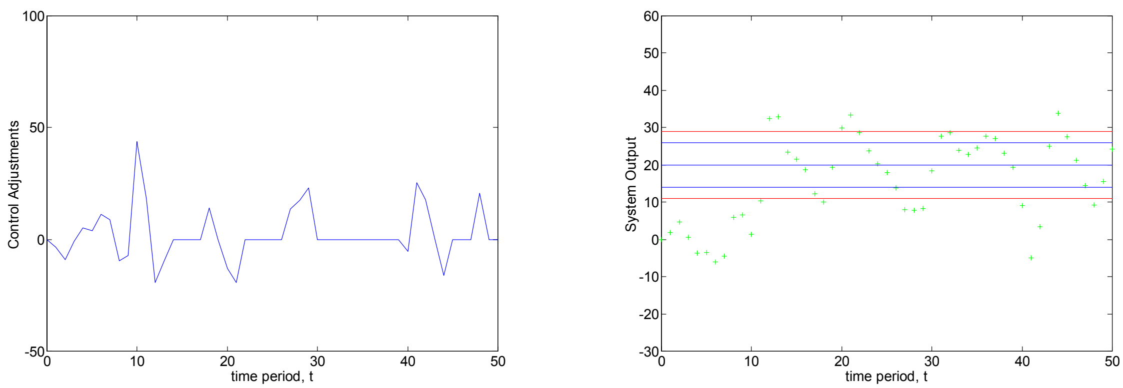

Figure 8. The IPC Mechanism is Designed to Activate the PID Controller Only when the Shewhart Monitor Signals an Out-of-Control Condition.

Case 4: MCEWMA/PID – Out-of-Control Activation

In Case 4, the IMPC mechanism has a MCEWMA monitor and a PID controller which is the combination used in Case 2. However, as in Case 3, the monitor regulates the PID controller by allowing it to make a process adjustment only when an observation is outside the monitoring limits. Table 4 shows a typical series of output observations, monitoring limits, and adjustments using this design. Figure 9 shows the sample path for one simulation run under this policy. After the step change at $t = 10$, six observations are outside the monitoring limits.

TIME	OBSERVATION	UCL	CL	LCL	ADJUSTMENT
0	0	0	0	0	0
1	1.7961	7.2877	0	-7.2877	0
2	5.8172	7.6668	0.3592	-6.9484	0
3	5.7437	8.9039	1.4508	-6.0022	0
4	4.6651	9.8361	2.3094	-5.2173	0
5	3.9360	10.3028	2.7805	-4.7417	0
6	-1.9302	10.5042	3.0116	-4.4809	0
7	-6.0656	9.6242	2.0233	-5.5777	-9.8270
8	-7.3699	8.3482	0.4055	-7.5372	0
9	-9.4309	7.0903	-1.1496	-9.3895	8.5767
10	-4.7826	5.7609	-2.8058	-11.3726	0
11	7.9449*	5.3432	-3.2012	-11.7456	27.2832
12	33.2210*	8.1636	-0.9720	-10.1075	-9.2568
13	45.1718*	19.5724	5.8666	-7.8391	-28.4245
14	27.9783	31.7557	13.7277	-4.3004	0
15	13.1643	35.0179	16.5778	-1.8623	0
16	1.3471	34.2713	15.8951	-2.4811	0
17	-4.7108	31.7833	12.9855	-5.8123	0
18	6.7698	28.8886	9.4462	-9.9962	0
19	20.8671	28.2726	8.9109	-10.4507	0
20	33.4899*	30.8978	11.3022	-8.2935	19.5791
21	52.9254*	36.3420	15.7397	-4.8626	-27.1774
22	51.8688*	46.5148	23.1768	-0.1611	-37.0837
23	35.3885	53.6802	28.9152	4.1503	0
24	38.5003	54.9272	30.2099	5.4926	0
25	39.9299	56.5868	31.8680	7.1492	0
26	37.2586	58.1939	33.4804	8.7668	0
27	31.2235	58.8518	34.2360	9.6202	0
28	25.8465	58.1425	33.6335	9.1245	0
29	18.3937	56.5739	32.0761	7.5783	0
30	18.5381	54.0579	29.3396	4.6214	0
31	22.1114	51.9862	27.1793	2.3724	0
32	20.2786	50.8951	26.1657	1.4364	0
33	16.7933	49.6570	24.9883	0.3196	0
34	17.1562	48.0171	23.3493	-1.3185	0
35	20.1783	46.7251	22.1107	-2.5037	0
36	24.5303	46.2221	21.7242	-2.7737	0
37	24.9986	46.6751	22.2854	-2.1042	0
38	21.9591	47.1091	22.8281	-1.4529	0
39	18.8101	46.8150	22.6543	-1.5064	0
40	9.1798	45.9527	21.8854	-2.1818	0
41	0.0518	43.5924	19.3443	-4.9037	0
42	0.5898	40.2968	15.4858	-9.3252	0
43	3.4805	37.5945	12.5066	-12.5813	0
44	5.1031	35.8099	10.7014	-14.4072	0
45	6.0526	34.6208	9.5817	-15.4574	0
46	9.3781	33.8120	8.8759	-16.0602	0

47	10.4552	33.7879	8.9764	-15.8352	0
48	8.9549	33.9633	9.2721	-15.4190	0
49	7.3170	33.7763	9.2087	-15.3589	0
50	5.8559	33.2814	8.8304	-15.6207	0

Table 4. A Typical Series of Output Observations, Monitoring Limits, and Adjustments Made in Case 4.
* Observation outside monitoring limits

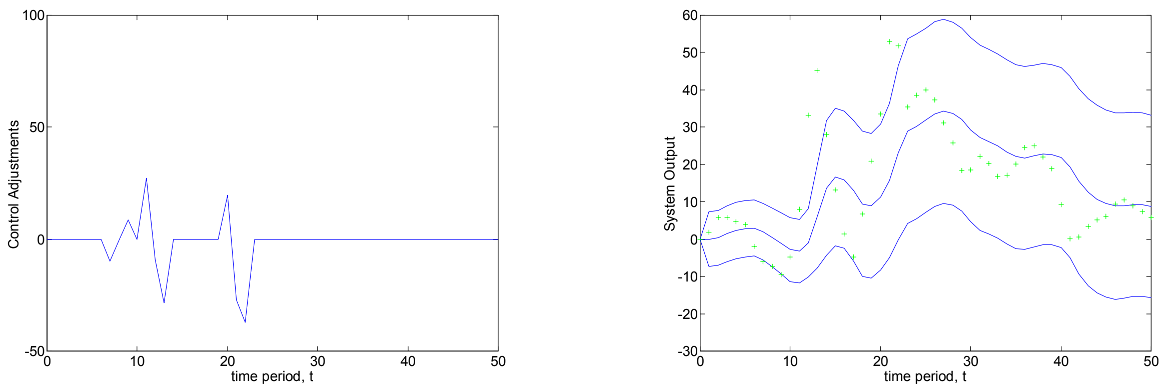

Figure 9. The IPC Mechanism is Designed to Activate the PID Controller Only when the MCEWMA Monitor Signals an Out-of-Control Condition.

Simulation Results and Discussion

The control policies in each of the four cases were evaluated on the sum of squared errors, number of adjustments, average adjustment, and number of alarms. In an industrial setting, the sum of squared errors and number of adjustments are statistics that would likely to be tied to cost and quality. The sum of squared errors measures the errors from the old and new target (i.e., before and after the step change in the input) so it we can view it as a measure of variation for the transition period. Note that the mean squared error statistic would have less relevance because of the trajectory involved in the transition phase. Of course, the quality control model will tell us to lower variation. However, in the type of system we are investigating, our results show that lowering the variation will nearly always result in increasing the number of adjustments (or conversely, decreasing the number of adjustments will increase variation). If the adjustment is rather easy to determine or can be implemented automatically then the cost for making it will be relatively small.

The average adjustment and number of alarms have less of an impact on cost but help our assessment of the IPMC designs. The average magnitude of adjustments indicates how aggressive the control action must be in order to compensate for the disturbance. In some cases, this range may be limited by the designer or by the physical equipment. The number of alarms indicates how many times the monitoring control limits are exceeded during the transition period. Table 5 shows the median and range for these performance measures based on 500 independent replications of the simulation model.

Control Mechanism Design	Sum of Squared Errors		No. of Adjustments		Average Adjustment		No. of Alarms	
	Median	Range	Median	Range	Median	Range	Median	Range
Case 1: Shewhart/PID *Simultaneous Activation*	1564	621 – 3864	50	(constant)	7.68	5.19 – 11.59	5.5	1.0 – 17.0
Case 2: MCEWMA/PID *Simultaneous Activation*	1564	621 – 3864	50	(constant)	7.68	5.19 – 11.59	2.0	4.0 – 8.0
Case 3: Shewhart/PID *Out-of-Control Activation*	3203	612 – 18355	21	10 – 30	6.40	1.55 – 11.75	12.0	1.0 – 21.0
Case 4: MCEWMA/PID *Out-of-Control Activation*	9967	1839 – 474286	6	1 – 14	3.86	0.18 – 30.88	4.0	0 – 13.0

Table 5. Performance Results for the System with Second-order Dynamics and ARIMA (1,1,1) Disturbance.

In Cases 1 and 2, the simultaneous activation cases, the statistics are identical for the squared error (median 1564, range 621-3864), number of adjustments (median 50, range constant), and average adjustments (median 7.68, range 5.19-11.59). The reason for this outcome is that these factors pertain only to the PID controller and since the monitoring does not effect the controller in these cases, the performance is the same. However, number of alarms signaled is a function of the monitoring method. It is consistent with expectations that the number of alarms using the Shewhart chart is higher than that for the MCEWMA chart because the latter is specifically designed for autocorrelated data, which is given by the ARIMA disturbance.

In Cases 3 and 4, the out-of-control activation cases, where the monitoring regulated the controller, the Shewhart/PID policy has a squared error that is one-third that of the MCEWMA/PID policy. In essence, the fact that the Shewhart signals more often actually turns out to be an asset in reducing the variation. In short, we are no longer solely interested in waiting for a statistically significant deviation to occur when our objective is to control the output response. This has been generally suggested by MacGregor (1988) and Box and Kramer (1992). The latter authors point out that "waiting for some monitoring criteria to become statistically significant before we adjust can produce a much larger mean squared error than would be obtained from a suitable adjustment scheme." The results of the current study support this new shift in philosophy. We note however, that the results will vary if different parameters are used to design the charts and or the controller.

A comparison of the median versus the range values of the sum of squared errors in Cases 3 and 4, suggests that there are some outlying runs where the policy does not perform as well. This effect was inversely correlated with fewer numbers of alarms. Basically, it suggests that when little help was offered by the monitoring charts, the variation tended to increase. This is also consistent with the higher variance of Case 4 with the MCEWMA procedure. Because the MCEWMA will tend to follow a shift without signaling for it, if offered less help with variation than the Shewhart procedure in Case 3. It does, however, result in fewer adjustments which may be a desirable tradeoff in some circumstances.

In all of the cases there may be a question of what to do about the process when the alarms are signaled. In the typical case of steady-state operations, the alarm indicates that either there is an assignable cause that should be searched out and removed or that there was a false alarm. Here, in the case of transition operations, these explanations may still hold true but it may be best to be more conservative about identifying which explanation holds at a particular time period because it is conceivable that the standard average run length performance of the charts may be affected. All of this suggests that there should be an opportunity to develop an improved chart specifically for the transition case that would better assist the control mechanism.

7 Summary and Conclusions

The contribution of this paper is in developing new ways to monitor and adjust dynamic processes during the transition period. A transition period typically occurs after an operating change in the process (e.g., startup, shutdown, grade changes, etc.). In a dynamic process, there is a response lag between the time of the change and the time the change is actually realized in the system. We were specifically concerned with investigating policies for a system with a second-order dynamic process and an ARIMA disturbance component. This system model is typical of many practical processes with a transition period and there is a significant need for better control in these systems. For instance, food and beverage and pharmaceutical manufacturers attribute a large portion of their out-of-specification product to this period.

We developed and tested four policies for this type of system that integrated PID feedback controllers and Shewhart or MCEWMA process monitoring charts. We implemented these policies using a SIMULINK simulation model and applied them in a case study of a liquid-tank system. The case study shows sample paths of the adjustments and output responses for each of the four policies. It also gives the actual data for the sample paths including time periods, observations, adjustment periods, adjustment magnitudes, and upper and lower monitoring limits. Five hundred replications of the simulation model for each of the policies were carried out to determine performance statistics.

Neither a PID controller nor Shewhart chart will give MMSE in second-order systems. However, using them together in an integrated process control policy is recommended for a complex system where minimum variance and/or number of adjustments is a concern. Furthermore, since PID control is not a variance-minimizing policy for second-order systems, the Shewhart limits will more likely be exceeded in this system. However, it turns out that this is an asset in reducing variation because we no longer wait for only a statistically significant deviation to occur. On the other hand, the MCEWMA chart results in fewer adjustments. This suggests that there should be an opportunity to develop an improved chart specifically for the transition case that would better assist the control mechanism, especially in a multiple decision criteria environment.

In addition to the specific integrated monitoring and control policies, this work makes two other broader contributions. First, we hope to further expand the common knowledge base between the control engineers and quality engineers that face these types of transitions in industry. Second, we hope that it will be clear that the software to implement the policies is easy to use and that it will become more common in the quality and statistics areas.

In future work, these models will be used to determine the effectiveness of integrated policies for higher-order dynamic systems with other types of disturbances and process upsets. Additionally, they will be compared to other models that incorporate economic concerns directly into the mechanism.

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